Asset Class Diversification and Delegation of Responsibilities between a Central Bank and Sovereign Wealth Fund*

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This paper presents a model comparing the degree of asset class diversification abroad by a central bank and a sovereign wealth fund. We show that if the central bank manages its foreign asset holdings in order to meet balance-of-payments needs, particularly in reducing the probability of sudden stops in foreign capital inflows, it will place a high weight on holding safer foreign assets. In contrast, if the sovereign wealth fund, acting on behalf of the Treasury, maximizes the expected utility of a representative domestic agent, it will opt for relatively greater holding of more risky foreign assets. We also show how the diversification differences between the strategies of the bank and sovereign wealth fund are affected by the government's delegation of responsibilities and by various parameters of the economy, such as the volatility of equity returns and the total amount of public foreign assets available for management.

JEL Codes: E52, E58, F30.

1. Introduction

In the aftermath of the financial crises of the late 1990s, many countries, particularly in emerging markets, have focused on the accumulation of official reserve assets. More recently, authorities have chosen to entrust more of their public foreign assets to sovereign

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wealth funds (SWFs). Private analysts put current sovereign wealth fund assets in the range of $2 to 3 trillion or even higher. This amount is projected to grow to as much as $13 trillion in the next ten years, an amount larger than the current global stock of foreign reserves of about $11 trillion at year-end 2012.[1]

There are several reasons for the growth of sovereign wealth funds. First, commodity price increases have swelled the sovereign asset holdings of commodity-exporting countries where the public sector controls commodity exports or heavily taxes the revenues earned by private commodity exporters. In some cases, the assets accumulated by SWFs are used as a financial stabilizer if commodity prices fall and depress tax revenue declines. In other cases, the establishment of SWFs, such as Norway's Government Pension Fund and Chile's Pension Reserve Fund, was motivated by the desire to transform concentrated exposure of public assets to volatile commodity prices into a more balanced and diversified global exposure, thereby protecting the income of future generations. A second factor behind the growth of SWFs is a byproduct of efforts by many emerging-market countries to accumulate large stockpiles of international reserves by running persistent current account surpluses, efforts that in many cases have led reserve holdings to exceed what is needed for purely prudential reasons (see Jeanne 2007; Aizenman 2008). The goal of diversifying these reserves into potentially higher-yielding assets has been pursued by transferring them from the control of the central bank to an SWF with the mandate to pursue financial strategies aiming at higher long-run returns. For example, China set up the China Investment Corporation (CIC) in

[1]Setser and Ziemba (2009) estimate that assets for major sovereign wealth funds totaled $2.1 trillion at year-end 2008 (including $700 billion in non-reserve assets of Russia and Saudi Arabia, but excluding roughly $300 billion in non-reserve assets held by Hong Kong and other smaller funds). Truman (2008) estimates that SWFs managed just under $3 trillion at the end of 2007 (he also identifies another $2.3 billion under management by sovereign pension funds). His higher figure is partly attributable to higher estimates for funds managed by Abu Dhabi. Forecasts of future global SWF assets depend on a myriad of factors, including the magnitude of current account imbalances, central bank demand for greater liquidity, and changes in commodity prices and asset returns. See Aizenman and Glick (2009). More recent estimates yield figures as high as $3.3 trillion for 2010, $3.8 trillion for 2011, and $4.2 trillion for 2012, excluding sovereign pension funds (Sovereign Wealth Fund Institute 2013).

2007 to manage more aggressively a portion of the country's more than $2 trillion in public foreign assets.[2]

The optimal portfolio management of public foreign assets necessitates a balancing of the goals and investment strategies of the central bank with those of the SWF. In most countries, the SWF operates in a fundamentally different manner than the central bank which primarily holds official foreign reserves for precautionary reasons, as liquidity concerns necessitate a short investment horizon and low risk tolerance. Consequently, central banks generally invest their foreign exchange reserves conservatively in safe and marketable instruments that are readily available to monetary authorities to meet balance-of-payments needs, particularly in responding to sudden stops of foreign capital inflows. In contrast, SWFs typically seek to diversify foreign exchange assets and earn a higher return by investing in a broader range of asset classes, including longer-term government bonds, agency and asset-backed securities, corporate bonds, equities, commodities, real estate, derivatives, and foreign direct investment.[3]

The governance structure of central banks and sovereign wealth funds also differs. Central banks typically operate with some degree of independence, subject to achievement of specific monetary policy mandates, such as achieving an inflation or exchange rate target or maintaining financial stability. In contrast, SWFs usually operate with more explicit portfolio-management goals, with their performance monitored more directly by government authorities. In Norway, for example, the Ministry of Finance is responsible for the management of the Government Pension Fund and has delegated responsibility for the operational management of the Fund's foreign assets to Norges Bank Investment Management (NBIM), a separate

[2]Aizenman and Glick (2009) present statistical analysis supporting stylized facts about sovereign wealth funds. Evidence is provided about the association of SWFs with a country's fuel export performance and current account surpluses as well as the degree to which measures of SWF governance and transparency compare with national governance standards.

[3]There are exceptions to this division of goals. In some countries the central bank itself actively manages a portfolio with risky as well as safe assets. For example, Saudi Arabia's Monetary Authority manages the government's oil surpluses as well as riskier foreign investments without the vehicle of a stand-alone SWF. In China, the State Administration of Foreign Exchange (SAFE) has shifted its holdings towards riskier foreign investments, even with the establishment of the CIC.

part of Norway's central bank. Presently, the Chilean central bank delegates responsibility for making investment decisions and day-to-day running of the country's sovereign wealth fund to a financial committee that reports to the Chilean Finance Minister, who in turn reports to the country's President.

The growth of assets under foreign sovereign management by both central banks and sovereign wealth funds has prompted concerns about their motivations, particularly as they have diversified more into equity and other privately issued assets. In this paper we analyze the optimal patterns of portfolio diversification by a central bank and a sovereign wealth fund, given their specific policy mandates assigned by the government and the government's mode of delegating these policy responsibilities. In doing so, our focus is not on assessing whether actual foreign reserves are optimal or excessive per se, but rather to illustrate analytically the determinants of the portfolio-management behavior of central banks and sovereign wealth funds.

We start with a benchmark model with which we compare the optimal degree of diversification between safe and risky foreign assets depending on the authorities' utility function. We assume that the goal of the central bank is to maintain financial stability by focusing solely on the goal of reducing the probability of sudden capital stops. In contrast, we presume that the SWF, acting on behalf of the Treasury, maximizes the expected utility of a representative domestic agent.[4] In this framework we show that the central bank's focus on financial stability implies that it will place a higher weight on limiting the downside risk of sudden stops than will the SWF. Consequently, the bank tends to bias its portfolio strategy towards holding more of the safe foreign asset, whereas the SWF opts for greater diversification towards riskier foreign assets. We also show how the diversification gap between the strategies of the bank and the SWF is affected by various parameters of the economy, such as the volatility of equity returns and the total amount of public foreign assets available for management.

We follow with a more elaborate model of joint decision making in which we treat the Treasury as the dominant player and assume

[4]We abstract from the possibility that SWF investments may be driven by non-economic considerations. Concern that they may be has prompted a range of responses in potential recipient countries (see Aizenman and Glick 2007).

that it sets the total level of foreign assets managed by the central bank but delegates to the central bank the responsibility for maintaining financial stability through the composition of its portfolio. Conditional on the actions of the central bank, the Treasury then decides what level of public foreign assets to entrust to the central bank for management as well as the composition of the residual share of the country's public foreign assets managed through the SWF. In these circumstances, we find that the assignment of the financial stability objective to the central bank tends to increase the gap between the optimal diversification patterns of the bank and the SWF, with the central bank specializing even more in holding safe assets so as to minimize the downside risk of sudden-stop crises, while the SWF specializes more in holding foreign equity assets in its portfolio.

Our paper is related to several strands of the literature on central banking policymaking, capital flows, and financial crises in emerging markets. Emerging markets are particularly vulnerable to sudden stops and accompanying financial instability because of either limited ability to borrow in terms of domestic currency and/or limited access to a global lender of last resort.

One strand of the literature explains how central bank demand for foreign reserves may arise from precautionary motives under these circumstances, taking into account the opportunity costs of holding reserves and adjustment costs of using them (Flood and Marion 2002). More recent analyses focus on how holding reserves may mitigate the output effects of sudden stops and capital flight shocks. Aizenman and Lee (2007), for example, illustrate how in an economy where liquidity shocks stemming from sudden stops may force costly liquidation of long-term projects and loss of output, hoarding reserves can reduce liquidation costs and raise welfare.[5] Bianchi, Hatchondo, and Martinez (2013) model how reserves

[5]One may ask why domestic households or private banks, rather than the central bank, do not choose to accumulate foreign reserves, inasmuch as they also have reason to mitigate the effects of sudden stops. An explanation is that as long as private residents act as price takers, they will treat the interest rate and the probability of sudden stops as exogenous to their actions, curbing their incentive to internalize the impact of reserves on the probability and severity of sudden stops. This in turn implies a public finance role for the central bank in managing the reserves for a country characterized by "original sin" that is unable to borrow abroad in hard currency (see Aizenman and Marion 2004).

can reduce output costs associated with the rollover risk of foreign borrowing. Jeanne (2007) and Jeanne and Ranciere (2011) provide a utility-based estimation and calibration of the optimal level of reserves, determined by the trade-off between the consumption-smoothing benefits of reserves in the event of a sudden stop and their opportunity costs.

The notion that higher international reserves can reduce the expected costs of sudden stops goes back to the lessons of the financial crises of the 1990s (Feldstein 1999). Calvo, Izquierdo, and Loo-Kung (2012) find that economies with higher reserves experienced lower probabilities of sudden stops and lower output costs for a sample of 110 countries over the period 1992 to 2004. Calvo, Izquierdo, and Loo-Kung (2012), as well as Frankel and Saravelos (2010) and Dominguez, Hashimoto, and Ito (2012), find evidence that countries with higher reserves before the 2008–9 global financial crisis experienced milder contractions in economic activity during the crisis. These observations are consistent with the International Monetary Fund (IMF) Survey of Reserve Managers, which indicates that building a buffer for liquidity needs is the most frequently cited rationale for reserve accumulation by central banks (IMF 2011). For this reason we choose to characterize central bank behavior in the simplest framework by posing its objective as the management of reserves to minimize the costs of sudden stops. While we specify these costs solely in terms of the effect of foreign asset holdings on the probability of sudden stops, as we discuss below, it is fairly straightforward to extend the analysis to take into account the output costs of sudden stops, conditional on their occurrence.

Of course, the optimal reserve level indicated by any particular model is typically quite sensitive to the values of key parameters, such as the degree of risk aversion and the probability of a sudden stop. Indeed, in many countries the central bank holds stocks of foreign exchange reserves so large that it is hard to reconcile them with the objective of preventing a sudden stop. In fact, there is a literature emphasizing other objectives that a central bank may wish to pursue in addition to the goal of financial stability. For example, Dooley, Folkerts-Landau, and Garber (2003) focus on the undervaluation of the exchange rate as a motive for reserve accumulation as part of an export-led growth strategy. Benigno and Fornaro (2012) model how the government uses foreign exchange reserves to internalize growth

externalities present in the tradable sector as well as to provide liquidity to the corporate sector during periods of financial stress faced by the tradables sector. However, empirical work by Aizenman and Lee (2007) finds that mercantilist effects on the level of reserves, as captured by variables like export growth and deviations from purchasing power parity, play a smaller role than variables associated with a precautionary demand for reserves. Hence, we abstract from these other possible objectives of central bank behavior in our analysis.[6]

Another strand of the literature related to our paper addresses the role of government policy in managing capital flows, usually justified by some economic distortion or externality. In Bacchetta, Benhima, and Kalantzis (2011), for example, domestic residents are credit constrained in foreign markets, giving policymakers a rationale to improve welfare through policies such as capital controls and reserve accumulation. In Bianchi, Hatchondo, and Martinez (2013), the justification for capital flow management comes from externalities associated with the private sector not taking into account the effect of their actions on international interest rates and the credit constraint they face.

The rationale for government management of long-term saving through sovereign wealth funds or other public institutions is a related concern.[7] Possible public finance reasons for government-sector involvement in managing long-run savings include control challenges facing consumers with dynamically inconsistent preferences, implying a motive for consumers to constrain their own future choices. In these circumstances, the public sector may act as an agent, inducing the necessary commitment technology. Alternatively, households may have a shorter horizon than the government, inducing the public sector to save for future generations.

[6]More recent evidence is consistent with the possibility that countries engage in competitive hoarding of reserves for mercantilist reasons in order to "keep up with the Joneses" (Cheung and Qian 2009). "Hoarding wars" can lead to excessive reserves, ultimately creating greater demand to channel more foreign assets from the central bank to the SWF. We refrain from modeling this behavior as well.

[7]Similar issues have been raised regarding the role of national social security systems as mechanisms to foster intergenerational saving.

Norway's sovereign wealth fund, the Government Pension Fund, provides an example of these considerations. According to Norges Bank Investment Management (NBIM), which manages the fund, it "seeks to take advantage of its outlook and considerable size to generate high returns and safeguard Norway's wealth for future generations." The NBIM also states concerning the management of international reserves that "the reserves shall be available for use as part of the conduct of monetary policy with a view to promoting financial stability."[8]

The organization of the paper is as follows. Section 2 presents the basic model of asset management by a central bank and sovereign wealth fund. Section 3 analyzes the implications of joint management through delegation of policy goals. Section 4 considers an extension to the intertemporal case. Section 5 presents conclusions.

2. The Base Model

Central banks and sovereign wealth funds hold and manage public foreign assets typically accumulated through commodity export revenues or foreign exchange intervention in the face of current account surpluses. In this section we assume each government authority manages its own portfolio of foreign reserves and equities in accordance with its specific objective function and determines the diversification of a given initial level of public foreign asset holdings, A, between risk-free international reserves, R, and risky foreign equities, F, such that $A = R + F$.

2.1 Theoretical Specification

We start by treating the stock of foreign assets A as exogenous, but later show how it can be determined endogenously. International reserves yield a *gross* risk-free interest rate r, whereas equity yields a stochastic return $r+e$, where e is the equity premium. The realized

[8]See http://www.nbim.no/en/investments/investment-strategy/ and http://www.nbim.no/en/our-organisation/about-us/foreign-exchange-reserves-/ (accessed July 3, 2014). The transparency of the objectives and management of Norway's SWF and central bank is not accidental: Norway's Government Pension Fund Global topped the governance ranking of fifty-three SWFs in thirty-seven countries by Truman (2008).

value of public foreign assets at the end of the period, A_1, is given by

$$A_1 = rR + (r + e)F = rA + eF; \quad 0 \leq F \leq A. \tag{1}$$

The timing of the model is such that *after* (i) determination of the portfolio composition $\{R, F\}$ at the beginning of the period and (ii) the realization of the equity-return shock e at the end of the period, there is a probability ϕ of a sudden-stop crisis. Consistent with empirical evidence, we assume that the probability of a sudden stop depends negatively on the public sector's end-of-period total foreign assets, A_1, and positively on the exogenously given level of the country's aggregate foreign liabilities, B, such that $\phi = \phi[A_1/B]; \phi' < 0$.[9] (In our notation we denote arguments of a function within square brackets.) Note that greater holdings of foreign risky assets can raise the probability of a sudden stop in the case of *negative* equity-return shocks, i.e., $d\phi/F = \phi'[A_1/B](e/B) > 0$ if $e < 0$. In these circumstances, greater diversification towards foreign equities entails growing balance sheet vulnerability because adverse equity-return shocks *reduce* the value of foreign asset holdings, thereby raising the probability of sudden stops.[10]

In the absence of a crisis, the foreign debt B is repaid and (exogenous) end-of-period output Y_1 is received.[11] With the occurrence of a

[9]Frankel and Saravelos (2010), Calvo, Izquierdo, and Loo-Kung (2012), and Dominguez, Hashimoto, and Ito (2012) all find evidence that economies that had more reserves experienced smaller output losses associated with sudden stops.

[10]Foreign liabilities may be sovereign or private if the government serves as a lender of last resort in the event of a sudden-stop crisis. We can motivate this setup by assuming in the background of the model that agents finance long-term investment via banks, while banks intermediate short-term deposits into longer-term investment à la Diamond and Dybvig (1983). Financial integration implies exposure to sudden stops and capital flight shocks, which may drain the liquidity of the domestic banking system. A sudden stop entails costly premature liquidation of long-term investments, thereby reducing the output associated with these projects. The sudden-stop probability can be derived endogenously by allowing for liquidity shocks z to foreign borrowing (see Aizenman and Lee 2007). If $z < A_1$, the liquidity shock is financed by A_1, without disrupting long-term investment. If $z > A_1$, it leads to a sudden-stop crisis associated with premature and costly liquidation of $z - A_1$ units of capital.

[11]Most formal analyses of optimal reserve management that model reserve accumulation as a cushion against external shocks and capital flow reversals treat the level of foreign borrowing debt as given and abstract from willingness-to-pay

crisis, however, the debt is not repaid and there is a penalty P associated with premature liquidation costs and possibly punitive actions by lenders that is assumed to be proportionate to end-of-period output and foreign assets:[12]

$$P = \tau(Y_1 + A_1), \quad 1 \geq \tau > 0.$$

Hence consumption without ("ns") and with ("ss") a sudden stop is, respectively,

$$C_1^{ns} = Y_1 + A_1 - B,$$
$$C_1^{ss} = Y_1 + A_1 - P.$$

Agents are assumed to be risk averse, with utility function U. The SWF determines its portfolio allocation in order to maximize the expected utility of a representative domestic agent, V_{sw}:

$$V_{sw} = E\left[(1 - \phi[A_1/B])U\left[C_1^{ns}\right] + (\phi[A_1/B])U\left[C_1^{ss}\right]\right], \qquad (2)$$

subject to the foreign asset accumulation relation (1), where E is the expectation operator. Assuming an interior solution with

concerns associated with sovereign debt. One notable exception is Alfaro and Kanczuk (2009), who analyze the joint decision by a sovereign to hold foreign debt and reserves. They show that holding reserves reduces the amount of sustainable debt by reducing the cost of exclusion from the capital market following default. In a calibrated version of their model, they also show that it may be optimal policy not to hold reserves at all. However, Bianchi, Hatchondo, and Martinez (2013) note that the presence of short- and long-term debt in a multiperiod model modifies Alfaro and Kanczuk's result by providing reserves a role in allowing the government to hedge against future increases in borrowing cost and rollover risk.

The model here can be generalized by endogenizing the determination of foreign borrowing B. For example, in a two-period economy, where second-period output is subject to productivity shocks, the possibility that the country may default in the second period with a bad-enough productivity shock implies that foreign borrowing is constrained by the condition that the cost of funds equals the expected return to lenders (see Aizenman and Marion 2004). Y_1 might be expressed as a function of borrowing B in this more general framework.

[12]This is a reduced form of the resources left following the premature liquidation of investments needed to finance deposits and capital flight. To simplify, we assume that A_1 and Y_1 shrink at the same rate. The logic of our analysis continues to hold with different shrinkage rates of A_1 and Y_1, where the resources left would be $A_1(1 - \tau_A) + Y_1(1 - \tau_y), 1 \geq \tau_A > 0, 1 \geq \tau_y > 0$.

diversification, the first-order condition determining F for a given level of A is

$$dV_{sw}/dF = E\left[(1-\phi)U'\left[C_1^{ns}\right](e) + \phi U'\left[C_1^{ss}\right]e(1-\tau)\right] - E\left[\phi'[A_1/B](e/B)\left\{U\left[C_1^{ns}\right] - U\left[C_1^{ss}\right]\right\}\right] = 0. \quad (3)$$

Rearranging terms,

$$E[dU/dF] = E[(\phi'[A_1/B])(e/B)\left\{U\left[C_1^{ns}\right] - U\left[C_1^{ss}\right]\right\}], \quad (4)$$

where

$$E[dU/dF] \equiv E[(1-\phi)U'\left[C_1^{ns}\right](e) + \phi U'\left[C_1^{ss}\right]e(1-\tau)].$$

The left-hand side of (4), $E[dU/dF]$, is the expected marginal benefit of diversification, i.e., the expected marginal utility valuation of the equity-return premium associated with higher F, $dA_1/dF = e$. The right-hand side of (4) is the expected marginal cost of diversification in the presence of a negative equity premium, i.e., the marginal increase in the probability of a sudden stop, $d\phi/dF = \phi'[A_1/B](e/B)$, times the utility cost of lost consumption associated with a sudden stop, $U[C_1^{ns}] - U[C_1^{ss}]$.[13] Thus, the SWF increases its holdings of foreign equity assets to the level that balances the marginal increase in returns with the marginal cost of a rise in the probability of a sudden stop.

In contrast to an SWF, central bank (CB) management is assumed to focus solely on maintaining financial stability, minimizing the probability of a crisis. The political economy of financial supervision may imply that there are benefits to delegating authority to an independent agency in order to overcome the temptation to relax supervision in "good" times (see Goodhart 2008). Moreover, the financial crisis of 2008–9 illustrated that controlling inflation by itself does not alleviate the central bank's need to focus on financial stability as an independent objective.[14] Accordingly, we refrain from modeling the determination of inflation and abstract from other possible central bank objectives, such as targeting inflation under

[13]Note also that the utility cost is positive as long as $(A_1 + Y_1)\tau > B$, i.e., the penalty costs are greater than the foregone foreign borrowing obligation.

[14]See Nier (2009) for further discussion of the importance of financial stability among the objectives of a central bank.

a flexible exchange rate regime.[15] Instead, we simplify by specifying the CB's objective function as minimization of the expected probability of a sudden stop

$$V_{cb} = E[\phi[A_1/B]]. \tag{5}$$

We abstract here from other factors affecting the cost of sudden-stop crises, such as the decline in the associated fall of output.[16]

Consequently, the first-order condition from maximizing (5) with respect to the CB's level of foreign equities F, while subject to (1), is

$$dV_{cb}/dF = E[-\phi'[A_1/B](e/B)] = 0. \tag{6}$$

That is, the central bank increases its foreign equity assets to a level that minimizes the probability of a sudden stop by setting its marginal impact on the probability to zero.

Comparing the first-order condition for optimal portfolio diversification by the SWF, (4), with that for the CB, (6), reveals that the CB ignores the possible expected gains to domestic agents induced by diversification in the presence of a positive equity premium. Instead, the CB minimizes the downside risk associated with adverse equity shocks, recognizing that such shocks would increase the leverage ratio B/A_1 at a rate proportionate to the equity portfolio share (since (1) implies $A_1/A = r + e(F/A)$), thereby increasing

[15]The post-crisis resumption of large financial inflows to emerging markets provides another impetus for focusing on financial stability as the main objective of central banks in affected countries. We also do not model deeper reasons for our assignment of objectives, such as a possible bias towards nominating a conservative central bank governor with a narrower policy focus than the representative agent (see Rogoff 1985; Walsh 1995; Obstfeld 1996; and Aizenman and Glick 2008).

[16]It is straightforward to redefine the central bank's objective to be minimization of the expected *cost* of sudden stops, defined by the probability of a sudden stop *times* the associated output loss conditional on the occurrence of a sudden stop (see Calvo, Izquierdo, and Loo-Kung 2012). If the conditional costs of a sudden stop are exogenously fixed, there is no effect on the optimal policy of the central bank. If the conditional costs are endogenously related to foreign asset holdings, equation (5) can be modified to include a multiplicative term involving A_1/B. In this case, if higher asset holdings reduce the output costs, an additional term would appear in the first-order expression (6) for dV_{cb}/dF, reflecting the central bank's gain associated with the lower output loss during crises, acting to increase the gain from hoarding foreign reserves.

the probability of sudden stops in bad times.[17] This suggests that the CB will opt for much less diversification into foreign equities than the SWF.[18]

2.2 Simulation Results

Further insight into the diversification patterns of the CB and SWF is gained by considering a simple simulation, with parameter values chosen to illustrate the challenges facing East Asian and commodity-exporting countries—accumulating significant foreign assets at levels that frequently exceed their external liabilities, while also being exposed to the risk of debilitating sudden stops and capital flight crises. We assume constant relative risk aversion (CRRA) utility where the relative risk aversion parameter β has a typical value of 2. The equity premium e equals an expected equity premium ε plus an equity-return shock δ that takes on two values, either $\bar{\delta}$ or $-\bar{\delta}$, each with probability one-half, implying mean value 0 and standard deviation $\sigma_\delta = \bar{\delta}$. We assume a gross safe interest return r of unity and an expected equity premium of 5 percent, i.e., $r = 1, \varepsilon = 0.05$.[19] We normalize initial output to $Y_1 = 1$, initial foreign assets A to 0.5, and foreign borrowing B to 0.5. The experiences of Korea, Chile, and other emerging markets with ratios of foreign borrowing to GDP around 50 percent and ratios of foreign reserve holdings to foreign borrowing fluctuating around 100 percent suggest that these parameter values are reasonable benchmarks.[20] We assume a sudden-stop crisis penalty (τ) of 20 percent, in line with the range of 10–20 percent reported by Hutchison and Noy (2006). Lastly, we

[17]Note B/A_1 is proportional to A/A_1 since $B/A_1 = (B/A)(A/A_1)$ and B/A is constant.

[18]Of course, alternative objective functions imply different trade-offs in the determination of optimal foreign reserve holdings. For example, Jeanne and Ranciere (2011) assume that the government maximizes the utility of domestic agents in a framework in which the risk of a sudden stop is the only source of uncertainty. They find that optimal reserve accumulation depends on the degree of risk aversion of domestic agents as well as on foreign borrowing, the interest rate, and the output costs of sudden stops. In our base model, only the SWF's portfolio depends on risk aversion, since the central bank objective function does not depend on domestic utility.

[19]Assuming a small positive net interest return affects the result trivially.

[20]See Bank of Chile (2010, chart 8) for details of Chile's experience and Jeong (2009) for an overview of Korea's experience.

specify the probability of a sudden stop as a polynomial function, $\phi[A_1/B] = (B/(A_1 + B))^h/[(B/(A_1 + B))^h + n]$, where the parameter values $h = 4$ and $n = 0.25$ are chosen to generate reasonable variations in the probability of sudden stops and changes in the composition of asset holdings as we vary other parameters in the simulation. This choice of parameter values gives sudden-stop probabilities in line with the empirical estimates of Furceri, Guichard, and Rusticelli (2011) and Jeanne and Ranciere (2011).[21]

The left panel of figure 1 plots the optimal share of foreign equities in a portfolio managed by the SWF (top curve) and the CB (bottom curve), each as a function of volatility $\bar{\delta}$, for our base case with $A = 0.5$.[22] Observe that a lower return volatility $\bar{\delta}$ raises the optimal share of foreign holdings, F/A, for both a central bank and a sovereign wealth fund. In addition, because of the difference in preferences, the sovereign wealth fund holds more equities for any given level of volatility than does the central bank. To illustrate, for $\bar{\delta} = 0.3$ (depicted by the vertical line), the SWF's equity share in its portfolio, F/A, is 100 percent (point a), whereas the CB opts for a much lower equity share, about 33 percent (point b).[23]

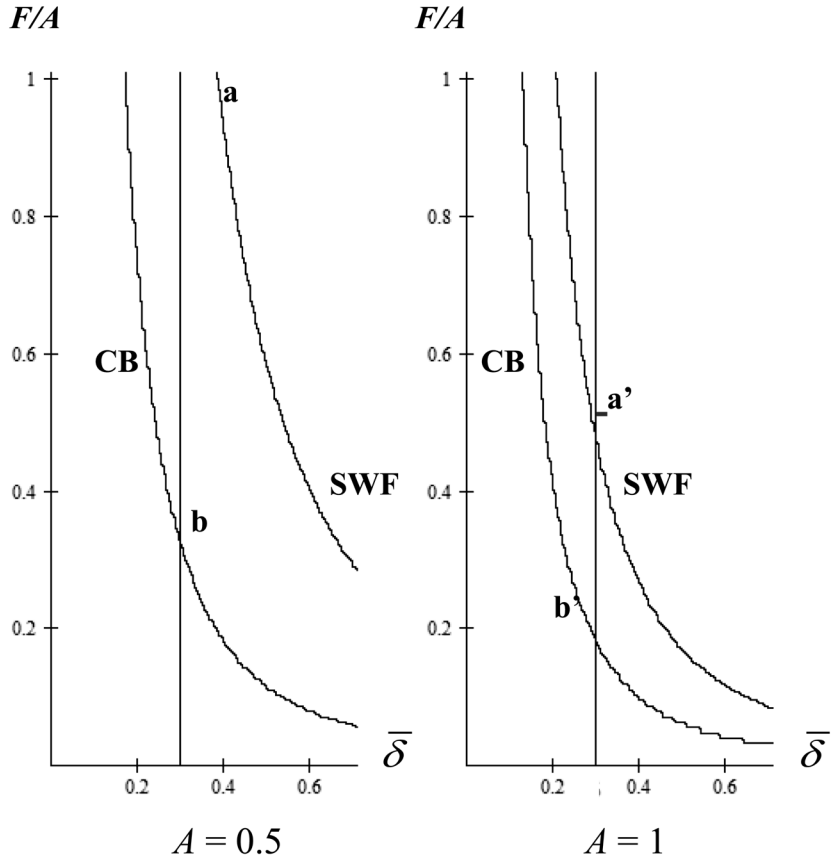
Changes in parameter values affect the optimal composition of each institution's portfolio. For example, a higher level of volatility in equity returns, say $\bar{\delta} = 0.4$, induces both institutions to reduce their foreign equity holding. The SWF would reduce its equity investment from 100 percent of its portfolio to about 94 percent, whereas the more conservative CB reduces its portfolio share of equities by a relatively greater amount, from about 33 percent to 18 percent. As another example, a lower degree of risk aversion by the domestic agents implies that the share of equity assets held by the SWF is higher for any given level of volatility. As another example, a lower degree of risk aversion by domestic agents—i.e., lower β—shifts the SWF schedule up (not shown), implying that the share

[21] Jeanne and Ranciere (2011) estimate sudden-stop probabilities in the range of 5 percent to 14 percent and, in their calibration exercises, consider probabilities in the range of 0 to 25 percent.

[22] More specifically, these curves reflect the locus of points satisfying (3) and (6), respectively. The jagged edges of the curves in our figures reflect the discrete nature of the computer-generated solutions to the optimization problem.

[23] We assume that the SWF is prohibited from borrowing, hence its equity position is maximized at 100 percent.

Figure 1. Equity-Returns Volatility, Foreign Equity Holdings, and Foreign Asset Size

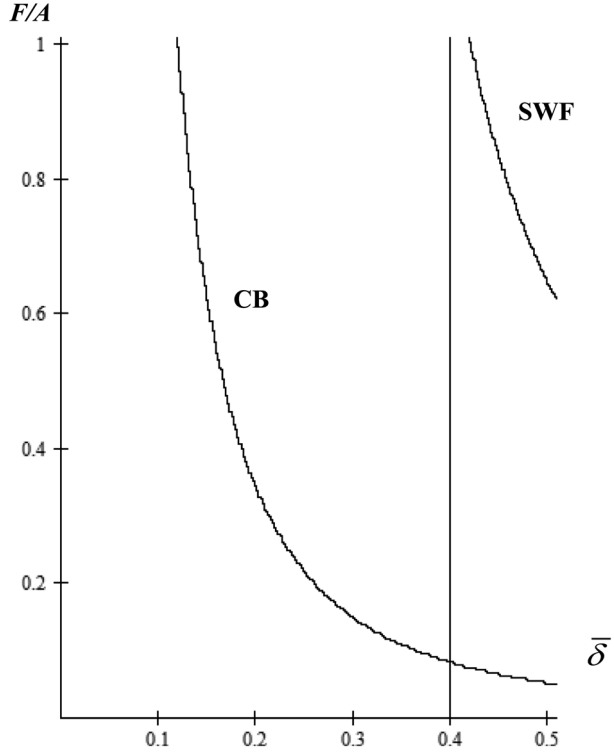


Notes: The bottom (top) curve describes asset diversification of the CB (SWF). The simulation assumes $U[C] = C^{1-\beta}/(1-\beta), \phi[A_1/B] = (B/(A_1 + B))^h/[(B/(A_1+B))^h + n]$, the equity shock δ takes on two values, either $\bar{\delta}$ or $-\bar{\delta}$, and $r = 1, \beta = 2, Y_1 = 1, A = B = 0.5, \varepsilon = 0.05, \tau = 0.2, h = 4, n = 0.25$. The left-hand and right-hand panels assume initial asset size levels A of 0.5 and 1, respectively. The vertical line corresponds to $\bar{\delta} = 0.3$.

of equity assets held by the SWF is higher for any given level of volatility.[24]

[24]The CB's portfolio is not affected since its objective is solely to minimize the probability of sudden stops, which does not depend on the private sector under our assumption that private borrowing is exogenous.

Figure 2. Equity-Returns Volatility and Portfolio Specialization Patterns: An Example



Note: The simulation assumes the equity shock δ takes on two values, either $\bar{\delta}$ or $-\bar{\delta}$, and $r = 1, \beta = 2, Y_1 = 2, A = 0.5, B = 0.4, \varepsilon = 0.05, \tau = 0.1, h = 6, n = 0.25$.

In the previous examples, for a reasonable range of variation in the volatility of equity returns, both institutions are diversified and hold both the risky and the safe foreign asset. In many countries, however, with both a sovereign wealth fund and a central bank, the two institutions specialize their portfolio investment, with the sovereign wealth fund holding only risky assets and the central bank holding (almost) only safe foreign assets. We illustrate this case in figure 2, where the combination of assuming a lower crisis penalty ($\tau = 0.1$), lower foreign debt holdings ($B = 0.4$), and higher equity-return volatility ($\bar{\delta} = 0.4$) induces the SWF to specialize totally

into holding only foreign equity assets ($F/A = 1$), while the CB chooses to allocate more than 90 percent of its assets into safe foreign reserves. Our analysis suggests that for a country without an SWF initially, a rising foreign asset base A may increase the opportunity cost associated with the limited portfolio diversification of the CB sufficiently to induce the authorities to establish an SWF, acting as an agent for the Treasury in pursuit of greater returns.[25]

We next turn to analysis of the comparative statics implications for the probability of sudden stops. We begin by analyzing how the size of the initial public asset position A affects the probability of a sudden stop for fully specialized institutions, i.e., for a sovereign wealth fund investing only in risky foreign assets ($F/A = 1$) and for a central bank investing only in safe assets ($F/A = 0$).

Figure 3, left panel, plots the probability of crisis as a function of the foreign asset position A, for the case where all assets are allocated to equities. The lower curve, ϕ_l, depicts the crisis probability relation when the realized equity shock is positive, with a realized value of $\delta = 0.3$. The top curve, ϕ_h, depicts the crisis probability relation when the equity shock is negative, with a value of $\delta = -0.3$. The other parameters are set at the same values as in the benchmark case presented in figure 1.

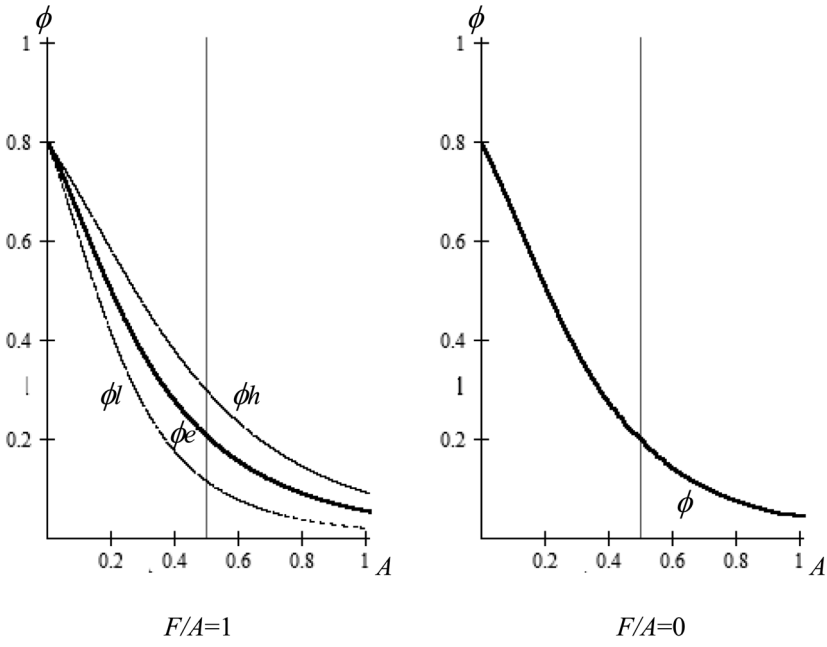
Observe that both curves are negatively sloped, as greater foreign asset holdings (relative to foreign borrowing) imply a lower probability of a sudden stop.[26] The ϕ_h curve lies everywhere above the ϕ_1 curve because for given initial foreign asset holdings, a negative shock results in a greater decline in the asset position, resulting in a higher probability of a crisis. The bold curve, ϕ_e, is the expected crisis probability (calculated as a weighted average).

The right panel of figure 3 describes the effect on the crisis probability for an institution, such as a central bank, holding only safe reserves. In this case the probability of a sudden stop still depends negatively on the level of public assets, but there is no exposure to equity return shocks. Thus greater diversification into equities

[25]This is consistent with the finding reported by Aizenman and Glick (2009) that older sovereign wealth funds not only have relatively high asset-to-GDP ratios but also have relatively low foreign reserve/GDP ratios compared with more newly established funds.

[26]Recall that foreign borrowing is treated as exogenous throughout our analysis.

Figure 3. Sudden-Stop Probabilities for Varying Initial Total Foreign Asset Holdings, with $F/A = 1$ (Left Panel) and $F/A = 0$ (Right Panel)

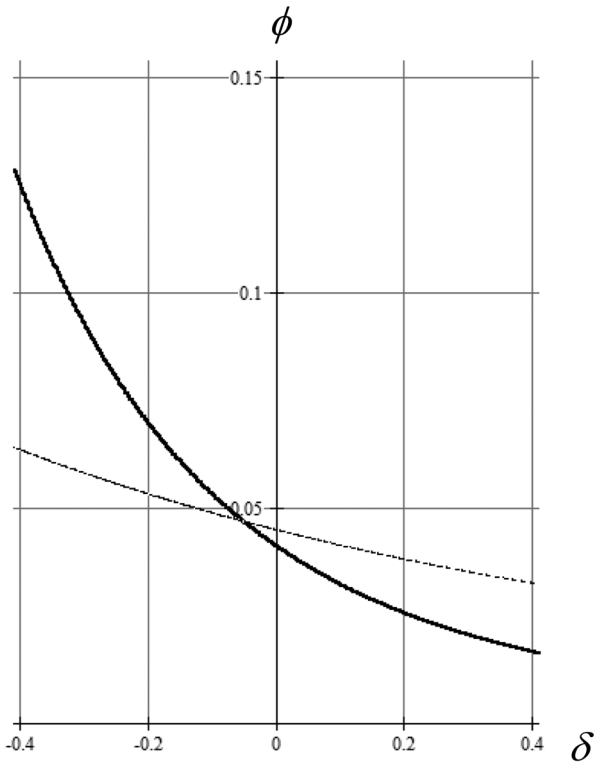


Notes: In the left panel, the lower curve, ϕ_l, is the crisis probability when the equity shock is positive, $\delta = 0.3$; the top curve, ϕ_h, is the crisis probability when the equity shock is negative, $\delta = -0.3$; and the bold curve, ϕ_e, is the expected crisis probability. In the right panel, the crisis probabilities are independent of equity shocks. The simulation assumes $U[C] = C^{1-\beta}/(1-\beta)$, $\phi[A_1/B] = (B/(A_1 + B))^h/[(B/(A_1+B))^h+n]$, and the equity shock takes on two values, either $\bar{\delta}$ or $-\bar{\delta}$.

increases exposure to equity shocks, with a second-order magnitude effect on the expected probability of a crisis. In comparison, an institution such as a central bank that opts for a much lower diversification into equities thereby reduces the downside risk associated with the exposure to bad realizations of equity return shocks.

How does the probability of a sudden stop depend on changes in the volatility of returns? Figure 4 plots the association between the equity shock realization δ and the sudden-stop probability ϕ for the CB (dotted line) and SWF (bold line), given the optimal equity

Figure 4. Probability of Sudden Stop for Different Equity-Return Shock Realizations



Notes: The bold curve describes the sudden-stop probability for $F/A = 1$ (the SWF's optimal equity share for a volatility level $\bar{\delta} = 0.3$), while the dotted curve shows the sudden-stop probability for $F = 0.33$ (the CB's optimal equity share for $\bar{\delta} = 0.3$). The simulation assumes the parameters specified in figure 1, left panel ($A = 0.5$).

share F chosen by each institution when $\bar{\delta} = 0.3, A = 0.5$, and all other parameters are the same as in the benchmark case depicted in figure 1. More specifically, these relations are determined by first determining the institution's optimal F conditional on $\bar{\delta} = 0.3$—call it F^* (which equals 100 percent of A for the SWF and 33 percent for the CB; see figure 1, left panel)—and then calculating the implied value of end-of-period assets $A_1, A_1 = rA + eF^*$, and hence the crisis probability $\phi = \phi[A_1/B]$ for varying realizations of the equity

premium $e = \varepsilon + \delta$, in turn corresponding to different realizations of δ (not necessarily equal to $\bar{\delta}$ or $-\bar{\delta}$).

The negative slopes of the curves plotted in figure 4 indicate that more positive return realizations imply higher end-of-period asset holdings and a lower probability of sudden stops for both the CB and SWF. Observe as well that the limited diversification strategy pursued by the CB results in a relatively low probability of sudden stops, less than 0.065, for the entire range of equity-return shocks shown (-0.4 to 0.4). In contrast, the greater holdings of risky foreign equities by the SWF expose the economy to the greater downside risk of a sudden stop in bad times—e.g., for a δ realization of -0.4, the probability of a crisis is about 0.13, twice as high as the probability for the CB for the same magnitude realization. This analysis implies that the central bank's diversification strategy tends to be relatively more resilient to unanticipated returns by maintaining a relatively low probability of sudden stop for a wide range of possible shocks. This suggests why it can be advantageous to assign portfolio-management independence to the CB in order to limit the probability of sudden stops. However, these gains come at the opportunity cost of lost returns on foreign assets, which rise with the volume of funds under management.

To summarize, the central bank's focus on financial stability implies that its portfolio share of risky foreign assets is lower and less sensitive to changes in return volatility and in size than that of a sovereign wealth fund.

3. Delegation of Financial Stability Objective to Central Bank

This section extends the base model, recognizing the possibility that the central bank has a comparative advantage in avoiding sudden stops and maintaining financial stability, whereas the SWF has a comparative advantage in managing risky portfolios. We continue to assume that the initial total foreign assets of the public sector A, and aggregate foreign borrowings B, are exogenously determined. But we now treat the Treasury as the dominant decision maker, determining the split of the given level of public foreign assets A between the management of the SWF and the CB.

Each entity manages its own portfolio of foreign reserves and equities. Denote by A_{cb}, A_{sw} the initial assets designated for management by the SWF and CB, respectively, such that $A = A_{cb} + A_{sw}$. Note that if A is given and the Treasury determines A_{cb}, then A_{sw} is determined as well.

The CB diversifies its portfolio between foreign reserves (R_{cb}) and foreign equities (F_{cb}), given its total initial asset holdings $A_{cb} = R_{cb} + F_{cb}$, implying that the realized value of assets under the CB's management at the end of the period, $A_{1,cb}$, is

$$A_{1,cb} = rR_{cb} + (r + e)F_{cb} = rA_{cb} + eF_{cb}; \quad 0 \leq F_{cb} \leq A_{cb}. \qquad (1')$$

As before, r is the (gross) risk-free rate and $e = \varepsilon + \delta$ is the stochastic equity premium for investment in risky assets by the central bank, with ε the expected equity premium and δ a mean-zero equity-return shock.

The SWF is assumed to have a comparative advantage in active management of the external portfolio, resulting in a higher expected equity premium $\varepsilon_{sw}, \varepsilon_{sw} \geq \varepsilon$, for any given volatility δ. This reflects the possibility that the CB diversifies by passive portfolio investment channels (such as investing in broad and relatively liquid foreign stock indexes), whereas the SWF diversifies by more active portfolio management, including foreign direct investment, etc., possibly offering higher expected returns. Accordingly, the realized value of assets under SWF management at the end of the period is $A_{1,sw}$:

$$A_{1,sw} = r(A - A_{cb}) + e_{sw}F_{sw}; \quad 0 \leq F_{sw} \leq A_{sw} \leq A. \qquad (1'')$$

As the CB deals directly with monitoring and supervising financial intermediation, we presume that the CB has a comparative advantage in reducing the probability of a financial crisis triggered by sudden stops. This may reflect the lower liquidity of the SWF's investments as well as institutional frictions that may slow down the SWF's ability to mobilize resources. Specifically, for a given ratio of assets to foreign borrowing A_1/B, the CB's management results in a lower probability of sudden stops than would occur under SWF management, i.e., $\phi_{cb}[A_1/B] < \phi_{sw}[A_1/B]$ for all A_1/B, where $\phi_{cb}[\cdot], \phi_{sw}[\cdot]$ denote the sudden-stop probability if assets are managed by the CB or a Treasury-controlled SWF, respectively.

The cost of delegating financial stability management to the CB is that, in case of need, only a fraction q of the SWF's assets A_{sw} would be available to the CB ($0 \leq q < 1$). Hence, under CB management $\phi_{cb}[\cdot] = \phi_{cb}[(A_{1,cb} + qA_{1,sw})/B]$. For simplicity, we assume that $q = 0$, implying $\phi_{cb}[\cdot] = \phi_{cb}[A_{1,cb}/B]$. As before, the probability of a sudden stop is assumed to depend negatively on the ratio of the public sector's foreign assets to foreign liabilities, implying $\phi'_{cb} < 0$.

3.1 Delegation and Diversification

The diversification problem facing the CB is identical to the one described in section 2, i.e., choose the level of foreign equity assets F_{cb} that maximizes the objective function

$$V_{cb} = E[(1 - \phi_{cb}[A_{1,cb}/B])U[C_{cb}] + (\phi_{cb}[A_{1,cb}/B])U[C_{cb}(1 - \tau_{cb})]] \tag{2'}$$

subject to the asset accumulation relation (1'), where ϕ is replaced with ϕ_{cb}. We continue to treat the foreign debt overhang B as given by assumption and note that A_{cb} is exogenous to the CB's actions, since it is determined by the Treasury. An analogue to the first-order condition (6) holds:

$$dV_{cb}/dF_{cb} = E[(-\phi'_{cb}[A_{1,cb}/B])(e/B)] = 0. \tag{6'}$$

As in the case without delegation, the CB sets policies that minimize the probability of sudden stops. Note that its optimal level of reserves, determined by condition (6'), implies that the expected equity premium evaluated by its impact on the probability of a sudden stop is zero.

The SWF's expected utility is akin to (2), modified by delegating the financial stability objective to the CB:

$$V_{sw} = E[(1 - \hat{\phi}_{cb}[A_{1,cb}/B])U[C^{ns}_{1,sw}] + (\hat{\phi}_{cb}[A_{1,cb}/B])U[C^{ss}_{1,sw}]], \tag{2"}$$

where

$$C^{ns}_{1,sw} = Y_1 + A_{1,sw} - B,$$
$$C^{ss}_{1,sw} = (Y_1 + A_{1,sw})(1 - \tau)$$

and $\hat{\phi}_{cb}$ is the probability of a sudden stop, conditioned on optimal CB policy as determined by (6').[27]

The Treasury's problem is to determine, for a given initial level of total public foreign assets A, the optimal levels of the SWF's investment in equities F_{sw} and the CB's initial assets A_{cb} that maximize its expected utility (2"), subject to (1"). The first-order condition for optimal F_{sw} holdings by the SWF, assuming an internal equilibrium (i.e., $0 < F < A_{sw}$), is

$$E[dU_{sw}/dF_{sw}] \equiv E[[(1-\hat{\phi}_{cb})U'[C^{ns}_{1,sw}] + (\hat{\phi}_{cb})U'[C^{ss}_{1,sw}](1-\tau)](e_{sw})] = 0. \quad (4")$$

Comparing (4") to (4), we infer that delegating financial stability policy to the CB implies that the SWF equates the expected marginal benefit of diversification, $E[dU_{sw}/dF_{sw}]$, to zero, instead of, as in the no-delegation case, equating it to the expected cost of increasing the probability of a sudden stop. Note also that condition (4") implies that in an SWF's equilibrium, the expected marginal utility associated with the equity premium is zero.

The Treasury also determines the CB's initial foreign assets, A_{cb}, with the SWF managing the remainder, $A_{sw} = A - A_{cb}$. The discussion in the previous section suggests that an increase in A should lead to greater asset allocation by the Treasury to the SWF in order to secure higher returns. This holds trivially as the probability of a sudden stop, $\hat{\phi}_{cb}$, approaches zero. More formally, the first-order condition determining A_{cb} is

$$\begin{aligned} dV_{sw}/dA_{cb} = {} & E[((1-\hat{\phi}_{cb})(-U'[C^{ns}_{1,sw}]) \\ & + (\hat{\phi}_{cb})(-U'[C^{ss}_{1,sw}])(1-\tau))(r + e\hat{F}'_{cb})] \\ & - E[(\hat{\phi}'_{cb}((r+e\hat{F}'_{cb})/B)\{U[C^{ns}_{1,sw}] - U[C^{ss}_{1,sw}]\})] = 0 \end{aligned} \quad (7)$$

[27] To derive $\hat{\phi}_{cb}$, first note that (6') yields an expression for F_{cb} as an implicit function of A_{cb}, as well as B and e: $F_{cb} = \hat{F}_{cb}[A_{cb}; B, \varepsilon, \sigma_e]$, where $\partial\hat{F}_{cb}/\partial A_{cb} > 0$. Substituting $\hat{F}_{cb}$ into ϕ gives $\phi_{cb} = \phi_{cb}[A_{1,cb}/B] = \phi_{cb}[(rA_{cb} + \hat{F}_{cb}[A_{cb}]e)/B] \equiv \hat{\phi}_{cb}[A_{cb}; B]$, where for large-enough r and small-enough σ^2_e, $E[\partial\hat{\phi}_{cb}/\partial A_{cb}] < 0$. Note the latter follows from the approximation $E[\partial\hat{\phi}_{cb}/\partial A_{cb}] = E[\hat{\phi}'_{cb}(r + e\partial F_{cb}/\partial A_{cb})] \cong E[\{\hat{\phi}'_{cb}[0] + \hat{\phi}''_{cb}[0]e\}(r + e(\partial F_{cb}/\partial A_{cb}))] = \hat{\phi}'_{cb}[0]r + \hat{\phi}''_{cb}[0]\sigma^2_e(\partial F_{cb}/\partial A_{cb})$.

or

$$E[dU/dA_{cb}] = E[-\hat{\phi}'_{cb}e(\hat{F}'_{cb}/B)\{U[C^{ns}_{1,sw}] - U[C^{ss}_{1,sw}]\}] = 0,$$

where

$$\begin{aligned} E[dU/dA_{cb}] \equiv E[&(1-\hat{\phi}_{cb})U'[C^{ns}_{1,sw}] \\ &+ (\hat{\phi}_{cb})U'[C^{ss}_{1,sw}](1-\tau)(r+e\hat{\phi}'_{cb})] \end{aligned}$$

and $\hat{F}'_{cb}$ is the marginal impact of an increase in initial foreign assets A_{cb} on the CB's optimal foreign equity holdings.[28] Thus the Treasury allocates more initial foreign assets to the central bank up to the point that the expected marginal benefit from doing so, $E[dU/dA_{cb}]$, equals the marginal increase in the probability of a sudden stop, $d\phi_{cb}/dF_{cb} = -\hat{\phi}'_{cb}e(\hat{F}'_{cb}/B)$, times the utility cost of a sudden stop, $U[C^{ns}_{1,sw}] - U[C^{ss}_{1,sw}]$.

3.2 Delegation and Welfare

In this section we analyze the welfare gains from the policy of delegating foreign asset management to the CB and SWF as described above. The representative consumer cares only about utility associated with the net resources available for household consumption. (We presume that the compensation of the CB, C_{cb}, and of the SWF's managers are negligible portions of GDP, and hence too small to impact consumption per capita.) Therefore, for the representative consumer, consumption is determined by the total sum of the assets accumulated by both agencies, $A = A_{sw} + A_{cb}$.

The consumer's expected utility is

$$V = E[(1-\phi)U[C^{ns}] + \phi U[C^{ss}]], \tag{8}$$

where

$$\begin{aligned} C^{ns}_1 &= Y_1 + A_{1,cb} + A_{1,sw} - B \\ &= Y_1 + rA + eF_{cb} + e_{sw}F_{sw} - B \end{aligned}$$

[28]As noted in the previous footnote, (6') yields an expression for F_{cb} as an implicit function of A_{cb}, as well as B and e: $F_{cb} = \hat{F}_{cb}[A_{cb}; B, e_\delta]$.

$$C_1^{ss} = (Y_1 + A_{1,cb} + A_{1,sw})(1-\tau)$$
$$= (Y_1 + rA + eF_{cb} + e_{sw}F_{sw})(1-\tau).$$

It is assumed that the default penalty is large enough such that a sudden stop is associated with lower consumption, i.e., $C^{ns} > C^{ss}$. The claim below follows:

Claim. *Assume that (i) the CB has a comparative advantage in prudential regulation, as defined by the relation* $\phi_{cb} = (1-b)\phi_{sw}, 0 \leq b \leq 1$*, where the parameter b measures the extent to which the probability of a sudden stop is lower with the CB than with the SWF, and (ii) the SWF has an advantage in portfolio management in the form of a higher expected return to foreign equity investment, i.e.,* $\varepsilon_{sw} > \varepsilon$*. Then the direct gains from delegation increase with the comparative advantage of each agency.*

Proof. The claim flows from the observation that $dV/db > 0$, $dV/d(\varepsilon_{sw} - \varepsilon) > 0$, i.e., consumer utility is increasing in the magnitude of the CB's comparative advantage and in the SWF's investment equity premium.[29]

Intuitively, the case for delegation is stronger the greater are the efficiency gains associated with allowing each agency to focus on its comparative advantage. Quite trivially, in the absence of any distortions and gains from the proper division-of-management objective among the CB and the SWF, delegation could be welfare reducing, since the CB's preferences are biased towards "safety first" relative to that of the representative consumer. Henceforth we assume that the gain from the division of goals is powerful enough to induce higher expected utility under delegation.

We close this section with an example illustrating the welfare gains associated with delegation. The example is based on the simulation described in the right-hand-side panel of figure 1, with $A = 1$ and $\varepsilon = 0.05$, where in the absence of delegation the optimal level of F was 0.48, i.e., the authorities allocated 48 percent of foreign assets

[29] These results can be established by noting that $dV/db = [\phi_{sw}\{U[C^{ns}] - U[C^{ss}]\}] > 0$ and $dV/d\varepsilon_{sw} = E[(1-\phi)U'_{c^{ns}} + \phi(1-\tau)U'_{c^{ss}} + (d\phi/d\varepsilon_{sw})(1-\phi)\{U[C^{ns}] - U[C^{ss}]\}] > 0$.

into equities. This amount of diversification implies that the equity return in the bad state is −25 percent (since $e_l = \varepsilon - \bar{\delta} = .05 - .3 = -0.25$, where e_1 denotes the equity return in the low state), and the sudden-stop probability, $\phi_{\delta=-\bar{\delta}}$, is 6.5 percent.

Suppose that delegation induces gains from the division of goals. In this spirit, we assume that delegating risk monitoring to the CB reduces the probability of sudden stop for any given amount of diversification by 40 percent, implying $\phi_{cb} = (1-b)\phi_{sw}$, with $b = 0.4$. We also assume that the expected equity premium earned by the SWF over the central bank is 2 percent, so that $\varepsilon_{sw} = \varepsilon + 0.02 = 0.07$. Lastly, assume that with the delegation of prudential management to the CB, the authorities (i) allocate to the CB the same amount of foreign assets that were put into reserves in the absence of delegation (i.e., $A_{cb} = 0.52 \cdot 1$) and direct the CB to invest all these assets in reserves and (ii) allocate to the SWF the remaining assets, $0.48 \cdot 1$, all to be invested in equities (i.e., $F_{sw} = A - A_{cb} = 0.48$).[30] It can be shown that, in the presence of the gains from the division of management described above, delegation leads to welfare gains similar to increasing the initial endowment by about 0.4 percent, reducing the probability of sudden stop in the bad state of nature from 6.5 percent to about 4 percent.[31]

In general, lower gains from the division of tasks among the CB and SWF imply more modest welfare gains. For example, for a value of b of 0.2, rather than 0.4, the probability of sudden stops in the bad state declines from 6.5 percent to 5.1 percent (rather than to 4 percent), reducing the welfare gains from delegation from 0.4 percent of the initial endowment to 0.3 percent. Fundamentally, the gains from delegation of foreign asset management follow from the country's balance sheet exposure associated with its foreign borrowing in foreign currency. Hence, the value of b can be interpreted as reflecting all factors affecting this exposure.

[30]It can be verified that for $A_{cb} = 0.21$, the CB has a strong bias towards foreign reserves. Thus, the CB would opt to invest more in reserves even in the absence of the authorities' directive.

[31]This result is obtained by calculating the expected utility gain, deflating it by the marginal utility evaluated for a zero equity-return shock. This ratio provides the expected utility gain in terms of the equivalent rise in the initial endowment Y_1. Deflating this gain by the initial endowment translates the welfare gain to the equivalent percentage increase in the initial output, Y_1.

4. Extending the Model to the Intertemporal Case

In this section we extend the model to the intertemporal case, where the authorities maximize the two-period expected utility function

$$V = U[C] + \frac{1}{1+\rho} E[U[C_1]] \tag{9}$$

with C, C_1 denoting consumption in the initial and subsequent periods, respectively, and ρ the intertemporal rate of time preference. This extension allows us to derive the total size of the public foreign asset portfolio (A) endogenously. Under delegation, policies are determined by the CB and SWF as described in section 3, where the representative consumer cares only about the net resources supporting consumption.

The initial-period consumption, C, is output Y, net of "investment" in foreign reserves R and foreign equity assets F (we abstract from the effect of past foreign borrowing, the "debt overhang," on income flows in the initial period). Next-period consumption, C_1, depends on asset returns and the costs of a possible sudden stop as well as repayment of the debt overhang B. As before, foreign reserves earn a gross return of r and equity assets a return of $r + e$, where the equity premium e equals an expected equity premium ε plus a zero-mean equity-return shock δ. Hence, the intertemporal consumption conditions are

$$\begin{aligned} C &= Y - R - F \\ C_1 &= \left\{ \begin{array}{l} C_1^{ns} = Y_1 + rR + (r + \varepsilon + \delta)F - B \\ C_1^{ss} = Y_1 + rR + (r + \varepsilon + \delta)F(1 - \tau) \end{array} \right\}, \end{aligned} \tag{10}$$

where next-period consumption without and with a sudden stop are denoted by "ns" and "ss," respectively. We assume that the default penalty is large enough that a sudden stop is associated with lower consumption, i.e., $C_1^{ns} > C_1^{ss}$. Accordingly, the representative consumer's expected utility is

$$V = U[Y - R - F] + \frac{1}{1+\rho} E[(1 - \phi)U[C_1^{ns}] + \phi U[C_1^{ss}]]. \tag{11}$$

Applying the logic of section 3, with delegation such that $\phi = \phi_{cb}$, the CB acts to maximize consumer expected utility by minimizing

the probability of sudden stops.[32] The optimal levels of reserves and foreign assets that achieve this goal are determined by the conditions[33]

$$d\phi/dR_{cb} = E[\phi'_{cb}[A_{1,cb}/B]](r/B) = 0 \tag{12a}$$

$$d\phi/dF_{cb} = E[\phi'_{cb}[A_{1,cb}/B](r+e)] = 0. \tag{12b}$$

Note that in a diversified equilibrium, (12a) and (12b) jointly imply that the expected equity premium evaluated by its impact on the probability of sudden stop is zero:

$$E[\phi'_{cb}[A_{1,cb}/B](e/B)] = 0. \tag{13}$$

The SWF takes as given ϕ'_{cb}, the sudden-stop probability conditioned on optimal CB behavior, and determines its equity position F in a way akin to section 3. The optimality conditions for diversification by the SWF, i.e., holding positive levels of both F and R, are[34]

$$U'[C] = \frac{E[(r+e_{sw})[(1-\hat{\phi}_{cb})U'[C^{ns}_{1,sw}] + (\hat{\phi}_{cb})U'[C^{ss}_{1,sw}](1-\tau)]]}{1+\rho} \tag{14a}$$

$$U'[C] = \frac{rE[[(1-\hat{\phi}_{cb})U'[C^{ns}_{1,sw}] + (\hat{\phi}_{cb})U'[C^{ss}_{1,sw}](1-\tau)]]}{1+\rho}. \tag{14b}$$

Thus, in equilibrium the SWF equates current marginal utility to expected (discounted) future marginal utility. Note (14a) and (14b)

[32]This follows trivially from the assumption that the CB's manager is paid a lump sum in good times and only a fraction of it in sudden stops (possibly zero, losing his post).

[33]A sufficient condition for the central bank to hold both foreign equity and reserves is $d\phi/dF_{cb} = E[\phi'_{cb}[A_{1,cb}/B](r+e)/B]_{F=0} < 0$, i.e., adding foreign equity to the portfolio lowers the probability of a sudden stop.

[34]A sufficient condition for the SWF to hold reserves as well as foreign equity is if doing so increases expected utility evaluated at the optimal $F, \hat{F}_{sw}$, i.e., $rE[(1-\hat{\phi}_{cb})U'[C^{ns}_{1,sw}] + (\hat{\phi}_{cb})U'[C^{ss}_{1,sw}](1-\tau)]/(1+\rho)|_{\substack{F=\hat{F}_{sw}\\R_{sw}=0}} > U'[C]|_{\substack{F=\hat{F}_{sw}\\R_{sw}=0}}$ or, equivalently, through use of the first-order condition for optimal F, (14a), $E[\{(1-\hat{\phi}_{cb})U'[C^{ns}_{1,sw}] + (\hat{\phi}_{cb})U'[C^{ss}_{1,sw}](1-\tau)\}e_{sw}]/(1+\rho)|_{\substack{F=\hat{F}_{sw}\\R_{sw}=0}} > 0$.

imply that the expected marginal utility associated with the equity premium is zero:

$$E[e_{sw}[(1-\hat{\phi}_{cb})U'[C^{ns}_{1,sw}]+(\hat{\phi}_{cb})U'[C^{ss}_{1,sw}](1-\tau)]]=0. \qquad (15)$$

The equilibrium conditions above pin down the optimal levels of $F_{cb}, R_{cb}, F_{sw}, R_{sw}$. To illustrate, suppose that the configuration of parameter values is such that there is full division of management goals, with the CB holding only international reserves and the SWF holding only equities, i.e., $F_{cb}=0, R_{sw}=0$. This implies that $A_{cb}=R_{cb}, A_{sw}=F_{sw}$ and hence $A=R_{cb}+F_{sw}$. Hence the determination of R_{cb} and F_{sw} pins down A, the overall level of foreign assets under public management.

To gain further insight, we assume that the probability of a sudden stop follows a power function as specified in section 2, and that the risk-free real interest rate equals the subjective discount factor ($r=1+\rho$). It can be shown that the central bank's demand for reserves increases with the debt overhang, equity-return volatility, and the foreign equity held by the SWF, i.e., $R_{cb}=R_{cb}[B,\bar{\delta},F_{sw}]; R'_{cb,B}>0, R'_{cb,\bar{\delta}}>0, R'_{cb,F_{sw}}>0$. The equity holdings of the SWF decline with a higher debt overhang and greater volatility of equity returns, and increase with the expected equity premium, i.e., $F_{sw}=F_{sw}[B,\bar{\delta},\varepsilon_{sw}]; F'_{sw,B}<0, F'_{sw,\bar{\delta}}<0, F'_{sw,\varepsilon_{sw}}>0$.

What are the effects of changes in the exogenous parameters—$B,\bar{\delta},\varepsilon_{sw}$—on *total* public foreign assets? In the case of an increase in ε_{sw}, we can infer that the effect is unambiguously positive: an increase in the expected equity premium leads to higher F_{sw} (since $F'_{sw,\varepsilon_{sw}}>0$, which in turn leads to higher R_{cb} (since $R'_{cb,F_{sw}}>0$) as well. Intuitively, if the foreign equity return is higher, policymakers will want to increase the total stock of foreign assets ($A=R_{cb}+F_{sw}$) under public management.

The effects of an increase in foreign debt B or volatility $\bar{\delta}$ on public foreign asset holdings are ambiguous. For example, an increase in foreign borrowing B has the direct effect of reducing demand by the SWF for risky foreign assets F_{sw} (since $F'_{sw,B}<0$), which in turn indirectly reduces the central bank's demand for safe foreign assets R_{cb} as well (since $R'_{cb,F_{sw}}>0$). But there is also the direct effect that leads to higher R_{cb} in order to provide more insurance against sudden stops (since $R'_{cb,B}>0$). The overall sign depends on

which effect dominates. The effects of an increase in $\bar{\delta}$ have similar opposing effects.

5. Conclusions

The optimal portfolio management of public foreign assets necessitates a balancing of returns and risks, where the latter reflect the risks of financial instability as well as the volatility of investment returns. In this paper we have presented a model with which we compare the optimal degree of diversification into safe foreign reserve assets and other higher-yielding, but risky, foreign assets by a central bank versus that of a sovereign wealth fund. We show that if the central bank manages its foreign assets with the objective of reducing the probability of sudden stops in foreign capital inflows, it will place a high weight on the downside risk of holding risky assets abroad and will tend to hold primarily safe foreign assets for precautionary purposes. In contrast, if the sovereign wealth fund, acting on behalf of the Treasury, operates with the objective of maximizing the expected utility of a representative domestic agent, it will opt for relatively greater holding of higher-yielding, though riskier, foreign assets. We also show how the diversification differences between the strategies of the bank and the SWF are affected by the government's delegation of responsibilities and by various parameters of the economy, such as the volatility of equity returns and the total amount of public foreign assets available for management. In addition, we illustrate why it might be desirable for policymakers to delegate the objective of achieving financial stability to the central bank alone. An avenue for further research is to endogenize private borrowing through an appropriate collateral constraint and perhaps to add more interaction between the behavior of the CB and SWF with the borrowing decisions of private agents.

The global financial crisis of 2008–9 highlighted the importance of the precautionary purpose for holding international reserves by central banks. Moreover, the drop in global commodity prices and equity returns during the crisis reduced the supply of public foreign funds and dampened the relative appeal of investing abroad in risky assets by both central banks and sovereign wealth funds. If the volatility in financial markets remains high, monetary authorities may continue to place a high weight on holding more reserves as a

means of minimizing the expected costs of sudden-stop crises. Nevertheless, the resumption of global growth, particularly in emerging markets, appears to be restoring the attractiveness of foreign investment by sovereign wealth funds. Policymakers in these countries still must seek to balance the relative rewards of doing so against the gains from reducing the probability of financial instability.

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