Studies in Banking and Finance 3 (1986) 61–81. North-Holland

OPTIMAL FOREIGN BORROWING AND INVESTMENT WITH AN ENDOGENOUS LENDING CONSTRAINT*

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This paper analyzes optimal foreign borrowing and investment in a two-period certainty model with traded and non-traded goods. The model illustrates how borrowing and investment interact with relative prices to determine the intertemporal distribution of production and consumption. The paper also formulates an endogenous lending constraint tied to the costs and benefits of repudiation by the borrower. The incorporation of this lending constraint changes the pattern of resource allocation intertemporally and intersectorally in a manner depending on factor intensities in production, the intertemporal elasticity of substitution, and the extent of lenders' retaliatory threats.

1. Introduction

This paper analyzes intertemporal consumption optimization through investment and foreign borrowing within a small open economy that produces traded and non-traded goods. The main purpose of this paper is to investigate how these investment and borrowing decisions depend on interactions with the relative price of non-traded goods in terms of traded goods and on the existence of a lending constraint that arises from the possibility that international debts may not be repaid.

The role of relative non-traded goods prices in borrowing decisions has been addressed by Bruno (1976), Dornbusch (1983), and Martin and Selowsky (1984). They show how an intertemporal decline of the relative price of non-traded goods is necessary to generate the intersectoral resource shifts that accompany foreign borrowing. Such a decline over time induces the movement of factors out of the traded goods sector into non-traded production

*An earlier version of this paper, written while the first author was Associate Professor of Economics and International Business at New York University, was prepared with the support of the Country Policy Department of the World Bank as part of the Research Project on Country Foreign Borrowing Strategies, RPO-673-01. The views contained are those of the authors alone and do not reflect the views of the World Bank, the Federal Reserve Bank of San Francisco, or the Board of Governors of the Federal Reserve System.

when foreign borrowing is adding traded goods resources, and induces the reallocation of factors back into tradeables production when repayment obligations are due in the future. More recently, Razin (1984) has analyzed how real investment also affects relative price changes and factor movements between sectors.

Two-sector models of foreign borrowing and investment generally assume that the borrowing country may obtain an unlimited amount of credit at either a fixed or rising interest rate. The work of Eaton and Gersovitz (1981), Sachs and Cohen (1982), Sachs (1984), and Cooper and Sachs (1985), however, has pointed out that countries may be credit constrained when lenders believe there is a possibility that they may not repay their debt.[1] Positive lending then only arises when lenders are able to penalize borrowers in the event of repudiation and they limit overall credit to a level below the worth of this 'collateral'. In this paper we incorporate a lending constraint that follows in this spirit into a model with traded and non-traded goods. When the constraint is binding, optimal investment and borrowing are affected. More interestingly, we show how the relative price structure and consumption path are influenced and how accompanying intersectoral resource shifts are generated.

In section 2 we present the basic model using a two-period certainty framework and determine the equilibrium conditions for investment and foreign borrowing under the assumption that the borrowing country may obtain an unconstrained amount of credit at a fixed interest rate. The two-period assumption collapses many of the complex issues surrounding optimal growth paths, balance of payments stages, and the maturity of debt. It retains sufficient intertemporal richness, however, to provide insight into how investment and borrowing depend on and affect relative prices and into how these variables serve to expand production through capital accumulation, and to smooth consumption between sectors and across periods. While some of the results obtained in this section are similar to those derived elsewhere [e.g., Dornbusch (1983), Razin (1984)], their presentation facilitates comparison with those obtained in the presence of a lending constraint.

In section 3 we introduce a lending constraint which is tied to the lender's perception of the costs and benefits of default by the borrowing economy. The constraint is endogenous in the sense that these costs and benefits depend on the borrower's investment and borrowing decisions. The equilibrium conditions for investment and borrowing and the relative price structure and consumption path obtained when this constraint is binding are compared with those in the unconstrained case derived in section 2. We find

[1] Useful reviews of the recent literature on foreign borrowing models which recognize the possibility that borrowers may not repay can be found in McDonald (1982), Glick and Kharas (1986) and Saunders (1986).

that the constraint generally acts to push the relative price of non-tradeables down in both periods, but by proportionately more in the first, in order to shift resources into the production of traded goods at the time when borrowing is rationed.

We should emphasize that the assumption of certainty implies that we do not model actual repudiation. With full knowledge of the conditions affecting borrower behavior, lenders will only extend volumes of credit which they perceive borrowers will repay. Thus, the role of foreign borrowing as an insurance cushion against unanticipated future disturbances is not captured here.

Section 4 graphically depicts the determination of equilibrium borrowing and investment in both the unconstrained and constrained cases. We show that the impact of the credit constraint is generally to shift consumption of traded and non-traded goods from the present into the future by lowering the levels of both borrowing and investment. The impact on consumption in the non-traded sector depends on its relative factor intensity in production, while the effect on traded goods consumption depends on the intertemporal elasticity of substitution as well. Section 5 summarizes the results of the paper.

2. The model with unconstrained lending

This section formulates a two-period model of a small, open economy without a monetary sector that produces traded and non-traded goods (Q_{Tt} and Q_{Nt}, respectively, where $t=1,2$ indexes the time period). The small country assumption implies that the price of traded goods and the world interest rate may be treated as exogenous.

The factors of production, labor and capital, are mobile between sectors. In the first period, the availability of factors for use in production is determined by initial endowments (L_1, K_1). While the aggregate labor force is held fixed, physical investment I in the first period increases the capital stock in the second ($K_2=K_1+I$). We assume that only the traded good is invested but that capital is used in the production of both goods. There is no depreciation of capital between periods.

Within each period perfect competition amongst producers implies that factors are allocated across sectors to equate their value marginal products. This permits us to express output of each good as a function of the aggregate availability of factors and of the relative price of the non-traded good in terms of the traded good, p_t,

$$Q_{Nt}=Q_{Nt}[K_t,p_t], \qquad \partial Q_{Nt}/\partial K_t<0, \quad \partial Q_{Nt}/\partial p_t>0, \tag{1a}$$

$$Q_{Tt}=Q_{Tt}[K_t,p_t], \qquad \partial Q_{Tt}/\partial K_t>0, \quad \partial Q_{Tt}/\partial p_t<0. \tag{1b}$$

Because the aggregate labor force is held constant throughout the analysis, we suppress it in the supply equations (1a) and (1b) above. The output effects of an increase in the total capital stock at constant relative prices are given by the Rybczynski theorem. In the main discussion the traded-goods sector is assumed more capital intensive than the non-traded sector at all wage–rental ratios.[2] Hence, an increase in the capital stock leads to a decrease in non-traded goods output and an increase in traded goods production. We will, however, point out where the results are affected by a reversal in this assumption.

First-period consumption may be augmented by foreign borrowing. It is assumed that the country may borrow any amount D at a constant world interest rate, r, fixed in terms of traded goods, subject only to satisfaction of its intertemporal budget constraint. This implies that first-period borrowing must be followed by repayment of all principal and interest in the second period. We abstract until section 3 from the issue of what forces the country to repay its foreign debt in the second period.

The country's welfare maximization problem can be construed as choosing levels of consumption and investment $(C_{T1}, C_{N1}, C_{T2}, C_{N2}, I)$ that maximize discounted utility,

$$W = U_1[C_{T1}, C_{N1}] + U_2[C_{T2}, C_{N2}]/(1+\delta), \tag{2}$$

subject to the intertemporal budget constraint that total discounted expenditure equals discounted income (valued in terms of traded goods)

$$C_{T1} + p_1 C_{N1} + I + (C_{T2} + p_2 C_{N2})/(1+r)$$
$$- Q_{T1} - p_1 Q_{N1} - (Q_{T2} + p_2 Q_{T2})/(1+r) = 0, \tag{3}$$

and to the non-traded goods equilibrium conditions,

$$C_{N1} = Q_{N1}, \qquad C_{N2} = Q_{N2}, \tag{4a, b}$$

where C_{Tt}, C_{Nt} are consumption levels of traded and non-traded goods; $U_t[\ ]$ is a stationary utility function; and δ is the pure rate of time preference; $t = 1, 2$.

An equivalent maximization problem can be obtained by noting that (3) and (4) imply the following period budget conditions for tradeables:

$$C_{T1} = Q_{T1} + D - I, \qquad C_{T2} = Q_{T2} - (1+r)D. \tag{5a, b}$$

[2]The assumption that traded goods are relatively capital intensive is probably appropriate for most developing countries. See Kravis, Heston and Summers (1983).

In the first period consumption of traded goods is the sum of output plus the level of borrowing less the amount invested. In the second period consumption is output less debt service.

Substituting (4) and (5) in (2), the maximization problem can be construed as maximizing the following expression with respect to foreign borrowing, investment, and relative prices (D, I, p_1, p_2):

$$W = U_1[Q_{T1}[p_1] + D - I,\ Q_{N1}[p_1]] + U_2[Q_{T2}[I, p_2] - (1+r)D,\ Q_{N2}[I, p_2]]/(1+\delta), \tag{6}$$

where the first-period capital stock has been suppressed in the notation for output. We denote the optimal levels of the decision variables to this problem by D^*, I^*, p_1^* and p_2^*, respectively.

The first-order conditions imply (asterisks to denote the evaluation of variables at their optimal levels are used only where necessary)

$$U_{N1}/U_{T1} = -(\partial Q_{T1}/\partial p_1)/(\partial Q_{N1}/\partial p_1) = p_1, \tag{7}$$

$$U_{N2}/U_{T2} = -(\partial Q_{T2}/\partial p_2)/(\partial Q_{N2}/\partial p_2) = p_2, \tag{8}$$

$$(1+\delta)(U_{T1}/U_{T2}) = 1 + r, \tag{9}$$

$$\rho_2 = 1 + r, \tag{10}$$

where $\rho_2 \equiv \partial Q_{T2}/\partial K_2 + p_2\,\partial Q_{N2}/\partial K_2$ is the marginal change in the value of output in period 2 from a unit increase in investment, or equivalently, the return on capital evaluated in traded goods terms; and U_{Nt}, U_{Tt} denote the marginal utility with respect to consumption of non-traded and traded goods, respectively.

Conditions (7) and (8) give the familiar result that the marginal rate of substitution between non-traded and traded goods in each period should equal the relative price determined by the marginal rate of transformation. Condition (9) states that the marginal rate of substitution between present and future consumption of traded goods must equal the interest factor. Condition (10) requires equality between the marginal return to capital (in traded goods terms) and the interest factor. Note that from two-sector theory this return depends negatively on the relative price of non-traded goods in the second period, assuming this sector uses capital less intensively. Furthermore, it is independent of the aggregate capital stock and investment when production is non-specialized, given the relative price. Thus condition (10) implicitly determines p_2^*, the equilibrium relative price in the second period, as a function of the world interest rate (and any exogenous technological production shifts that affect the return to capital).

To derive more explicit results we adopt constant elasticity formulations of the utility function and output supply functions. Specifically,

$$U_t = C_t^{1-b}/(1-b), \qquad C_t \equiv C_{Tt}^{a} C_{Nt}^{1-a}, \qquad b \geqq 0, \qquad 0 \leqq a \leqq 1, \tag{11}$$

where C_t represents an index of aggregate consumption, and a is the share of traded goods. The parameter b is a measure of the concavity of the utility function $(-U_t'' C_t / U_t')$ and $1/b$ indicates the intertemporal elasticity of consumption substitution. As b tends to zero, $U_t[\ ]$ approximates a linear function, consumption shows greater intertemporal substitutability, and the incentive for consumption smoothing between periods diminishes. Conversely, as b tends to infinity, greater smoothing of consumption between periods is desired.

Using the specific functional form for utility (11), the first-order conditions can be written in more traditional form as[3]

$$(1+\delta)(C_2/C_1)^b = (1+r)(p_2/p_1)^{1-a} = \rho_2 (p_2/p_1)^{1-a}. \tag{12}$$

The left-hand side of (12) is the marginal rate of intertemporal substitution between aggregate consumption in the two periods, or alternatively the real social discount rate (RSDR), expressed in terms of the aggregate consumption bundle. Because of diminishing marginal utility of consumption the RSDR rises as consumption increases over time. The terms on the right-hand side represent the real cost of foreign borrowing and the real return to capital, respectively, also expressed in terms of the aggregate consumption bundle. Each depends on two elements: the first is the cost (return) in traded goods terms. The second adjusts this cost (return) by the change in the aggregate consumer price index, given by p_t^{1-a}. This indicates that changes in relative prices over time are important determinants of intertemporal substitution. If traded goods become more valuable over time $(p_2/p_1 < 1)$, the real cost of foreign borrowing, for example, will be raised.

The supply functions are specified as[4]

$$Q_{Nt} = (q_{Nt})(q_{Nt}/q_{Tt})^{e}(p_t)^{e}(K_t)^{-k}, \qquad e, k > 0, \tag{13a}$$

$$Q_{Tt} = (q_{Tt})(q_{Tt}/q_{Nt})^{e_T}(p_t)^{-e_T}(K_t)^{k_T}, \qquad e_T, k_T > 0, \tag{13b}$$

where q_{Nt}, q_{Tt} are shift parameters associated with Hicks-neutral technological change; e, e_T are output elasticities with respect to p_t; and k, k_T are output

[3]To derive (12), note that (11) implies $U_{Nt}/U_{Tt} = (C_{Tt}/C_{Nt})(1-a)/a$, $U_{T1}/U_{T2} = (C_1/C_2)^{1-b}(C_{T2}/C_{T1})$, which imply that (7) and (8) can be expressed as $(C_{Tt}/C_{Nt})(1-a)/a = p_t$, $t = 1, 2$.

[4]See the appendix

elasticities with respect to K_t; in the non-traded and traded sectors, respectively. Note that k and k_T are both positive under the assumption that production of non-traded goods is relatively labor intensive.

These specifications when applied to conditions (7), (8) and (9) enable us to solve for the following expressions for the relative price structure and relative consumption paths of traded and non-traded goods:[5]

$$p_2/p_1 = [(1+r)/(1+\delta)]^z (q_{N2}/q_{N1})^{-b(1+e)z} \times (q_{T2}/q_{T1})^{bez}[(K_1+I)/K_1]^{bkz}, \tag{14}$$

$$C_{N2}/C_{N1} = [(1+r)/(1+\delta)]^{ez} \rightarrow (q_{N2}/q_{N1})^{(1+e)z''z} \times (q_{T2}/q_{T1})^{-ez''z}[(K_1+I)/K_1]^{-kz''z}, \tag{15}$$

$$C_{T2}/C_{T1} = [(1+r)/(1+\delta)]^{(1+e)z}(q_{N2}/q_{N1})^{(1+e)z'z} \times (q_{T2}/q_{T1})^{-ez'z}[(K_1+I)/K_1]^{-kz'z}, \tag{16}$$

where

$$z \equiv 1/(1-a+b(a+e)) > 0,$$
$$z' \equiv (1-a)(1-b) \gtrless 0 \quad \text{as} \quad b \lesseqgtr 1,$$
$$z'' \equiv 1-a(1-b) > 0.$$

Eqs. (14), (15) and (16) describe how equilibrium changes in relative prices and relative consumption between periods depend on the world interest rate, rate of time preference, production shifts, and investment.

Consider first the case in which there is no investment ($I=0$), the case dealt with by Dornbusch (1983). In the absence of production shifts, the relative price structure depends only on the relation of the interest rate and the rate of time preference. If, for example, $r=\delta$, then p_2/p_1, C_{N2}/C_{N1}, and C_{T2}/C_{T1} are equal to one, and relative prices and consumption are constant over time. Intuitively, the marginal utility of present and future consumption of traded goods are equal; hence there is no incentive to borrow in order to shift consumption across periods. Since both the traded goods and non-traded sectors clear each period with the same equilibrium levels of consumption and production, p does not change from one period to the next.

If $\delta > r$, however, relative prices and consumption of both non-tradeables

[5]To derive (14), (15) and (16), note from reference to footnote 3 that $(C_{Tt}/C_{Nt})(1-a)/a = p_t$, $t=1,2$, and from the definition of aggregate consumption that $C_1/C_2 = (C_{T1}/C_{T2})^a (C_{N1}/C_{N2})^{1-a}$. Consequently, (9) can be expressed as $(1+\delta)(C_{T2}/C_{T1})^{1-a(1-b)}(C_{N2}/C_{N1})^{-(1-a)(1-b)} = 1+r$. Use of the non-traded goods equilibrium conditions (4) and the supply specification (13) leads to the results in the text.

and tradeables fall between periods, i.e., $p_2/p_1 < 1$, $C_{N2}/C_{N1} < 1$, $C_{T2}/C_{T1} < 1$. There is now an incentive to borrow in order to increase first-period consumption of both goods. However, because borrowing directly augments only the consumption of tradeables, the increased demand for non-tradeables induces a rise in p_1 and a shift of resources out of the traded sector and into the production of non-tradeables. In the second period when the debt is repaid, expenditures on both goods decline. However, since the debt repayment is in traded good terms, the relative price of non-traded goods falls in order to reallocate resources into the production of tradeables. Thus, borrowing permits the shifting of consumption into the first period, while the fall in p over time induces the reallocation of resources into the traded goods sector in the second period when the debt must be serviced. Note that this implies that the real cost of foreign borrowing exceeds the world interest rate. If $r > \delta$, these changes are reversed.

If $r = \delta$, but technological changes cause non-traded goods production to increase in period 2 relative to period 1 ($q_{N2} > q_{N1}$), then again $p_2/p_1 < 1$. While non-traded consumption unambiguously increases over time, the effect on relative consumption of tradeables depends on the magnitude of the intertemporal elasticity of consumption substitution, $1/b$. If $b < 1$ and consumption is relatively substitutable across periods, then (since $z' > 0$) $C_{T2}/C_{T1} > 1$; current consumption and borrowing are reduced in favor of the future. If $b > 1$, and intertemporal substitution is relatively low, then $C_{T2}/C_{T1} < 1$, and borrowing increases in the first period in order to smooth consumption across periods. The effects of traded goods production shifts are analogous.

Consider now the implications of positive investment. Investment in the model under consideration plays a role that differs distinctly from that of borrowing. While borrowing affects non-tradeable production and consumption only through relative prices, investment affects the non-traded sector directly through the influence of factor endowments on supply. If the non-traded sector is labor-intensive, higher investment reduces non-traded output in the second period. Thus, if $r \neq \delta$, or production shifts are unequal over time, investment as well as borrowing play roles in achieving intertemporal and intersectoral consumption smoothing.

Recall the previous example where $\delta > r$, borrowing is used to smooth consumption forward into the first period, and the relative price of non-tradeables rises in the first period, while falling over time. Investment partially offsets this change in relative prices. Intuitively, when there is an incentive to invest in the first period, any rise in p_1 is dampened in order to encourage the shift of resources out of the non-traded sector and into the production of the investment good, tradeable output. (Borrowing increases to dampen the fall in consumption of tradeables.) In the second period an excess demand for non-tradeables develops because the relative labor intensity of non-traded production leads to a decline in output when the capital stock expands. This induces an increase in the relative price which dampens the decline in the relative production of non-tradeables. Eq. (15) indicates that the production and consumption of the non-traded good fall unambiguously over time.[6] The effect on the relative consumption of tradeables again depends on the elasticity of substitution. If $b<1$, then $C_{T2}/C_{T1}<1$. When consumption is relatively substitutable across periods, the incentive to expand consumption along with tradeables production in the second period is weak. If $b>1$, the incentive to increase future consumption is strong and the consumption of tradeables rises over time.

Lastly, note that if $b=0$, as assumed by Razin (1984), relative price movements are independent of investment (as well as of production shifts). Consumption is then perfectly substitutable across periods, and the marginal rate of substitution between present and future tradeables consumption depends only on the rate of time preference and relative price movements. Consequently, investment or production shifts have no effect on the relative price structure.

We have yet to explicitly determine the equilibrium levels of the decision variables – D^*, I^*, p_1^* and p_2^* – which satisfy the first-order conditions, given by (7), (8), (9) and (10), and the budget conditions for traded and non-traded goods, given by (4) and (5). [Recall that p_2^* is determined by condition (10) alone.] We defer doing so until section 4, where a graphical solution to the optimal borrowing and investment problem is presented.

3. The model with constrained lending

In this section we drop the assumption that the borrowing country faces a perfectly elastic supply of foreign funds and incorporate an endogenous lending constraint. We follow the approach of Eaton and Gersovitz (1981), and Sachs and Cohen (1982) which supposes that borrowers choose to repudiate or reschedule when the gains of such an action outweigh the costs. The costs of repudiation arise from various retaliatory steps by lenders such as sanctions, trade embargoes, etc. In such an environment, lenders will limit the extent of the credit extended to a country to be below the worth of the 'collateral' given by the penalties to repudiation.

In our framework the benefits to repudiation in the second period are represented by the additional consumption of tradeable goods made possible by the non-repayment of the principal and interest amount due, $(1+r)D$. Following Sachs and Cohen (1982) we assume that the costs of default take the form of a given proportional loss v of tradeable goods production. Accordingly, the economy chooses to repay its debt only if the return to

[6]This result may be reversed if non-traded goods production is capital intensive.

repudiation is less than or equal to the cost, i.e., if

$$D \leqq vQ_{T2}[I, p_2]/(1+r). \tag{17}$$

Condition (17) may be interpreted as a lending constraint faced by the borrower. It implies that lenders will limit their supply of funds to the borrower according to the future level of output of the traded good, which in turn is a positive function of the level of investment and a negative function of the second-period relative price of non-traded goods. In other words, the greater investment in the first period or the lower the relative price of non-tradeables in the second, the greater is future traded goods output, and the greater the amount of funds that can be safely lent to the borrower with the confidence that these funds will be repaid.

When the borrower perceives the influence of its actions on the amount of credit forthcoming, the amount of investment undertaken takes into account not just the resulting expansion of second-period output and consumption, but also the effect of relaxing the credit ceiling. Accordingly, the country's welfare maximization problem becomes one of choosing D, I, p_1 and p_2 that maximize (6) while subject to (17).[7] Denote the optimal levels of the decision variables to this constrained maximization problem by D^c, I^c, p_1^c and p_2^c, respectively. The first-order conditions associated with this equilibrium imply (superscript c's are omitted)

$$U_{N1}/U_{T1} = -(\partial Q_{T1}/\partial p_1)/(\partial Q_{N1}/\partial p_1) = p_1, \tag{18}$$

$$U_{N2}/U_{T2} = -[(\partial Q_{T2}/\partial p_2)/(\partial Q_{N2}/\partial p_2)][1 + v\lambda(1+\delta)/(U_{T2}(1+r))]$$

$$= p_2[1 + v\lambda(1+\delta)/(U_{T2}(1+r))], \tag{19}$$

$$(1+\delta)(U_{T1}/U_{T2}) = 1 + r + (1+\delta)(\lambda/U_{T2}), \tag{20}$$

$$\rho_2 - (1+r) = (\lambda/U_{T2})(1 + r - v\rho_2)(1+\delta)/(1+r), \tag{21}$$

$$\lambda(D - vQ_{T2}/(1+r)) = 0, \tag{22}$$

where λ, the Lagrange multiplier corresponding to the debt constraint, may be interpreted as the marginal welfare gain of raising the economy's

[7] It is presumed that the borrower always carries out the investment plans made in the first period. This precludes the possibility that it announces a higher than planned investment level in order to secure additional credit from lenders who are willing to relax the debt ceiling on the basis of the announced plans. The absence of such cheating can be viewed as the result of previous games played by the borrowers and lenders over time. It is then in the interest of the borrower to cooperate because the penalty for cheating in any one game would have costs in terms of forgone benefits in future games.

borrowing capacity. λ is positive when the debt constraint is binding, and zero otherwise.

Observe from condition (18) that the constraint has no bearing on the relation between the marginal rate of substitution and the relative price of non-traded goods given by the marginal rate of transformation in the first period. In the second period, however, (19) says that the marginal rate of substitution exceeds the relative price of non-tradeables, implying that when the debt constraint is binding the economy consumes and produces relatively less non-tradeable goods. Note that the credit ceiling creates an externality that gives an added social gain to second-period traded goods production. There is, therefore, an optimal tax on non-traded goods (or subsidy to traded goods), given by $v\lambda(1+\delta)/(U_{T2}(1+r))$. Intuitively, $v/(1+r)$ gives the increase in the credit ceiling for a unit increase in traded output, λ measures the associated welfare gain, and $(1+\delta)/U_{T2}$ transforms this gain into second-period traded goods units.

Observe next, from (20), that the marginal rate of substitution between present and future traded goods consumption exceeds the interest factor. When the debt ceiling is binding, the country consumes more traded goods in the second period. Of course, this result in part directly follows from the limited ability to bring consumption forward into the first period by borrowing. Additionally, however, consuming relatively more tradeables in the second period raises the credit ceiling. The reason is that higher second-period consumption lowers the marginal utility of future consumption, and thus, lowers the welfare cost of making debt service payments and the incentive to repudiate.[8]

Recall that in the unconstrained case the return to capital in the second period, ρ_2, equalled the interest factor, $1+r$. Condition (21) indicates that the lending constraint drives a wedge between these two terms. To determine the relationship between ρ_2 and $1+r$ in this case note first that since λ/U_{T2} is positive, the two terms $\rho_2-(1+r)$ and $1+r-v\rho_2$ must be of the same sign. Since $v<1$, and therefore $\rho_2>v\rho_2$, these terms cannot be simultaneously negative. Consequently, $(1+r)/v>\rho_2>1+r$, and both terms are positive. Thus when the borrowing constraint is binding the return to capital exceeds the interest factor.

Since ρ_2 is a negative function of p_2, this result allows us to infer that, for a given world interest rate and technological conditions, p_2^c, the equilibrium relative price of non-tradeables in the second period when debt is constrained, is *less* than p_2^*, the price prevailing in the unconstrained case. Intuitively, when subject to a lending constraint linked to the production of

[8]Sachs and Cohen (1982), Sachs (1984), and Cooper and Sachs (1985) have also noted some of the effects of credit constraints on the equilibrium pattern of consumption. The models in these papers, however, ignore the role of non-traded goods and the critical effects of this constraint on relative prices.

second-period tradeables, it is optimal to lower the relative price of non-tradeables in order to shift resources into the production of traded goods. Observe also that the wedge between ρ_2 and $1+r$ diminishes the greater is the default penalty parameter v. For the greater is v and the perceived costs to default, the more debt creditors are willing to extend. Consequently, the difference between constrained and unconstrained debt levels is lower, and the welfare cost of the constraint is reduced.

As in the previous section, the constant elasticity specifications of the utility function (11) and output supply (13) can be utilized to obtain explicit expressions for the relative price structure and consumption profile. Observe first that rearrangement of (21) implies

$$\lambda(1+\delta)/U_{T2} = \phi(1+r), \tag{23}$$

where

$$\phi \equiv (\rho_2 - (1+r))/(1+r-v\rho_2) > 0.$$

Condition (23) relates the marginal welfare of relaxation of the borrowing constraint (in second-period tradeable goods terms) to the interest rate, to the wedge between the return to capital and the interest factor, and to the repudiation penalty parameter, v. Note that if the economy is unconstrained, $\rho_2 = 1+r$, and ϕ and λ equal zero.

Inserting (23) into (18) and (19) and utilizing (11) and (13) enables us to obtain the following results:[9]

$$p_2^c/p_1^c = \Omega(p_2^*/p_1^*)[(K_1+I^c)/(K_1+I^*)]^{bkz}, \tag{24}$$

$$C_{N2}^c/C_{N1}^c = \Omega^e(C_{N2}^*/C_{N1}^*)[(K_1+I^c)/(K_1+I^*)]^{-kz''z}, \tag{25}$$

$$C_{T2}^c/C_{T1}^c = (1+v\phi)\Omega^{1+e}(C_{T2}^*/C_{T1}^*)[(K_1+I^c)/(K_1+I^*)]^{-kz'z}, \tag{26}$$

where

$$\begin{aligned} \Omega &\equiv (1+\phi)^z/(1+v\phi)^{zz''} \\ &= [\rho_2/(1+r)]^z[(1+r)(1-v)/(1+r-v\rho_2)]^{a(1-b)z} > 1, \\ z &\equiv 1/(1-a+b(a+e)) > 0, \\ z' &\equiv (1-a)(1-b) \gtrless 0 \quad \text{as} \quad b \lessgtr 1, \\ z'' &\equiv 1-a(1-b) > 0. \end{aligned} \tag{27}$$

The superscript ‘c’ refers to the optimal values of variables in the constrained case, while the asterisk refers to those in the unconstrained case.

[9]See footnotes 3 and 5.

The impact of the lending constraint on the pattern of relative price movements and consumption smoothing of each good can be clearly seen from the above relationships. In general, there are two effects to consider.

The first reflects the effect of the lending constraint on relative prices in the absence of investment ($I=0$). Under this circumstance, (25) and (26) indicate that the lending constraint causes relatively more consumption of both goods to be postponed into the second period, i.e., $C^c_{N2}/C^c_{N1} > C^*_{N2}/C^*_{N1}$ and $C^c_{T2}/C^c_{T1} > C^*_{T2}/C^*_{T1}$, since $\Omega > 1$ and $1+v\phi > 1$. Higher second-period consumption of non-tradeables must be associated with a lower relative price. Hence we can conclude that $p^c_2 < p^*_2$. But we also observe from (24) that any decline in relative prices over time is reduced in the constrained case, i.e., $p^c_2/p^c_1 > p^*_2/p^*_1$.[10] Recall that in the unconstrained case when there is an incentive to borrow caused by (say) $\delta > r$, $p^*_2/p^*_1 < 1$. Foreign borrowing, by raising the supply of tradeables in the first period and reducing it in the second period, induces a fall in the relative price of non-tradeables over time. When borrowing is constrained, however, the movement in relative prices is dampened since there is then less of an incentive to shift resources into the non-traded sector initially and into the traded sector in the future. Note also that the two results that $p^c_2/p^c_1 > p^*_2/p^*_1$ and $p^c_2 < p^*_2$ imply $p^c_1 < p^*_1$. Thus, in the absence of investment, when borrowing is constrained in the first period the resulting excess demand for consumption of tradeables necessitates a rise in the price of tradeables and a decline in p_1.

The second effect of the lending constraint operates through the endogenous change in the equilibrium level of investment. As we shall demonstrate in section 4, investment in the constrained case is generally less than in the unconstrained case. The resulting impact on relative price movements and on intertemporal consumption smoothing depends on the factor intensity assumption, given by k, and on the magnitude of the intertemporal elasticity of substitution, $1/b$. When b is zero, the results above that $p^c_2 < p^*_2$, $p^c_1 < p^*_1$ and $p^c_2/p^c_1 > p^*_2/p^*_1$, still hold unambiguously. Production and consumption of non-traded goods unambiguously increase by more over time in the constrained case as long as non-traded goods are labor intensive ($k > 0$). A similar result for tradeables consumption holds if $b < 1$. These results have a straightforward explanation. When consumption is intertemporally highly elastic, the effect of investment on the consumption path of tradeables and non-tradeables has little direct impact on the relative price structure. The lower investment in the constrained case reduces the overall capital–labor ratio and hence favors more production and consumption of the labor-intensive non-traded goods in the second period. At the same time, it is associated with a proportionately

[10]Note that the divergence in the relative price structure between the constrained and unconstrained cases diminishes as e, the price elasticity of non-traded goods supply, increases (revealed by inspection of the parameter z in the coefficient Ω).

greater reduction in borrowing that, as long as consumption is relatively substitutable intertemporally, also serves to raise second-period tradeables consumption relative to the first-period level. With different assumptions about b and k the reverse result, that tradeables consumption rises less over time with a lending constraint, cannot be ruled out.

4. Optimal borrowing and investment

We now turn to determination of the optimal levels of foreign borrowing and investment for the unconstrained and constrained cases discussed in the previous sections. We depict these equilibria graphically, and show that in general the rationale for positive borrowing suffices to induce positive investment. We also show how the relationship between borrowing and investment depends crucially on the factor intensity parameter k, and on the repudiation penalty v.

We begin with the case of unconstrained borrowing. As discussed in section 2, optimal borrowing and investment are determined interdependently with the relative price structure, (p_1, p_2). We know, however, that since the return to capital, ρ_2, depends uniquely on p_2, the equilibrium second-period relative price, p_2^*, is in turn uniquely determined by the first-order condition (10) requiring equality between ρ_2 and the interest factor. Condition (10) thus yields the implicit relation

$$p_2^* = g[r], \qquad g' \lesseqgtr 0 \quad \text{as} \quad k \gtreqless 0. \tag{28}$$

When non-traded production is labor intensive, i.e., $k > 0$, then the return to capital depends negatively on the relative price of non-tradeables. Consequently, a higher world interest rate results in a lower equilibrium relative price of non-tradeables.[11]

To simplify matters we now adopt the assumption that the consumption elasticity parameter b is zero. As discussed in section 2, this implies that consumption is relatively substitutable across periods and that in the unconstrained case the relative price structure depends only on the interest rate and rate of time preference. More specifically, (14) reduces to

$$p_2^*/p_1^* = [(1+r)/(1+\delta)]^{1/(1-a)}. \tag{14'}$$

With p_2^* determined by (28), p_1^* is in turn pinned down by (14′).

Given the relative price levels p_1^* and p_2^*, the equilibrium levels of foreign borrowing and investment can be found by simultaneously solving the budget equilibrium conditions for the traded and non-traded sectors in both

[11] An expression for ρ in terms of p, and the technological shift parameters, q_N and q_T, which are suppressed here, is derived in the appendix.

the first and second periods together with the relevant first-order conditions. The first-period budget conditions (4a) and (5a), together with the marginal rate of substitution condition (7), imply[12]

$$(a/(1-a))p_1 Q_{N1}[p_1] = Q_{T1}[p_1] + D - I, \tag{29}$$

which implicitly defines the relationship

$$D = I + g_1[p_1], \qquad g_1' > 0. \tag{30}$$

Observe that to remain consistent with first-period budget conditions for a given level of investment, borrowing rises in response to an increase in non-tradeables production, and hence consumption, arising from a relative price shift towards that sector.

Analogously, for the second period, (4b), (5b) and (8) give

$$(a/(1-a))p_2 Q_{N2}[I, p_2] = Q_{T2}[I, p_2] - (1+r)D, \tag{31}$$

which yields the implicit relation

$$D = g_2[I, p_2, r], \tag{32}$$

$$\partial g_2/\partial I \gtreqless 0 \quad \text{as} \quad k \gtreqless 0, \qquad \partial g_2/\partial p_2 < 0, \qquad \partial g_2/\partial r < 0.$$

In the second period, higher borrowing must be associated with higher traded goods output and lower non-traded output. If $k > 0$, this comes about through higher investment. An increase in p_2 permits less tradeables consumption and hence the servicing of a lower level of debt. An increase in r, given I and p_2, implies less debt can be repaid in the second period.

In the constrained borrowing case, not only must optimal levels of borrowing and investment satisfy the budget conditions (30) and (32), but also the lending constraint condition (17). (17) defines the following relation

$$D = L[I, p_2, v, r], \tag{33}$$

$$\partial L/\partial I \gtreqless 0 \quad \text{as} \quad k \gtreqless 0, \qquad \partial L/\partial p_2 < 0, \qquad \partial L/\partial v > 0, \qquad \partial L/\partial r < 0.$$

The greater the level of investment or the higher the repudiation penalty, the greater the credit lenders are willing to extend to the borrower, knowing that it is in the self-interest of the borrower to repay these funds. A decrease in p_2 for any given level of I stimulates greater tradeables production in the

[12]Recall that with the utility specification (11), the first-order condition implies $(C_{T1}/C_{N1})(1-a)/a = p_1$.

second period and brings forth greater borrowing. A higher world interest rate, given I and p_2, results in less credit.

Condition (30) is depicted as the upward-sloping schedule $G1$ in fig. 1. Under the assumption that $k>0$, (32) and (33) are upward-sloping as well,

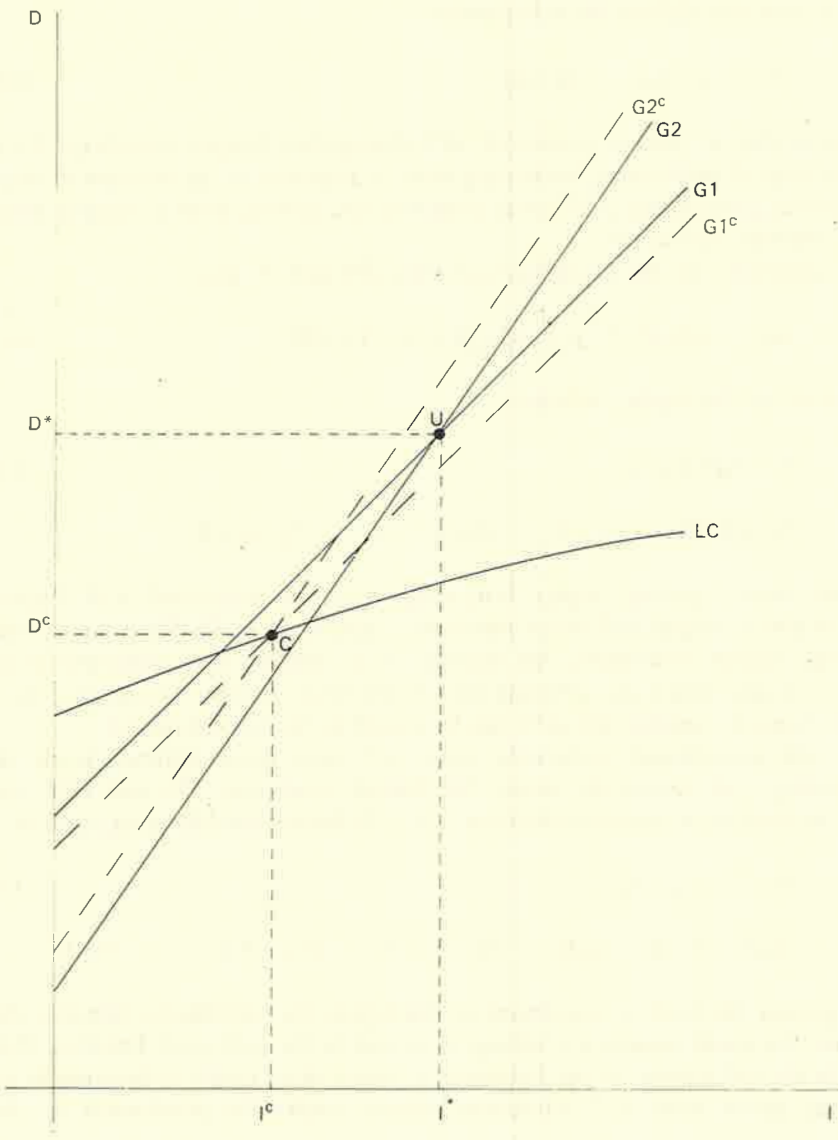

Fig. 1

depicted by the $G2$ and LC schedules, respectively, with the former steeper than the $G1$ curve. Intuitively, each unit of additional borrowing allows one unit of additional investment in the first period; thus $G1$ has a slope of one. In the second period, an additional unit of debt requires less than one unit of investment to permit repayment since the capital stock in the traded goods sector rises by more than the aggregate capital stock, if $k>0$; thus the $G2$ schedule has a slope greater than one. Note that the positions of the $G1$ and $G2$ curves depend on relative price levels. In the unconstrained case, the optimal relative price structure (p_1^*, p_2^*) pins down the positions of these curves. The corresponding intersection, denoted by point U in fig. 1, determines the optimal borrowing and investment levels, D^* and I^*.[13]

Only points on or below the LC curve in fig. 1 are feasible, since they correspond to levels of credit which lenders know will be repaid and thus will be willing to extend. Rational lenders will be unwilling to lend amounts corresponding to points above it since such amounts will not be repaid. The relative position of this schedule depends crucially on the repudiation penalty parameter, v, as well as on the second-period price of non-tradeables, p_2. If the magnitude of v is small,[14] then the LC curve will lie closer to the origin and fewer debt–investment combinations are feasible.

As drawn in fig. 1, the LC schedule lies below the intersection of the $G1$ and $G2$ curves, point U, implying that the unconstrained optimal borrowing and investment levels are unattainable. The constrained equilibrium depends on how the relative price effects of the binding lending constraint influence the $G1$, $G2$ and LC schedules. Recall from section 3 that when borrowing is constrained and b equals zero the relative price of non-tradeables is unambiguously pushed down in both periods, i.e., $p_2^c < p_2^*$ and $p_1^c < p_1^*$. These relative price effects imply that in the constrained case the $G1$ schedule shifts to the right (to $G1^c$), the $G2$ schedule shifts to the left (to $G2^c$) and the LC schedule shifts up (not shown). The new equilibrium is determined when all three schedules intersect, as at point C in fig. 1. The resulting equilibrium levels of borrowing and investment, D^c and I^c, are clearly below their unconstrained levels. Note also that the point C is below the $G1$ schedule, indicating that borrowing falls by more than investment when the constraint is binding.

These conclusions are quite sensitive to the factor intensity assumption. If the traded sector is labor intensive, then higher investment reduces traded output in the second period. Both the $G2$ and the LC schedules will then be

[13] The existence of an equilibrium is ensured as long as the vertical intercept of the $G1$ schedule exceeds that of the $G2$ schedule. A sufficient condition is $r<\delta$. Note also that the equilibrium depicted in fig. 1 is stable: below the $G1$ schedule, for given p_1, there exists an excess demand for tradeables in the first period which requires greater D and/or lower I; while above the $G2$ schedule, for given p_2, the excess demand for second-period tradeables requires either less repayment of D and/or greater I.

[14] For empirical evidence on this, see Kalitsky (1985).

negatively sloped. Consequently, the relative price of non-traded goods in the constrained case will be higher than in the unconstrained case, in order to satisfy condition (21) that the marginal return to capital exceed the world interest rate when credit is rationed. Thus, the $G2^c$ curve lies below the $G2$ curve (fig. 2). Clearly, the impact of the constraint is still to unambiguously lower borrowing, but investment may be either higher, lower, or unchanged.

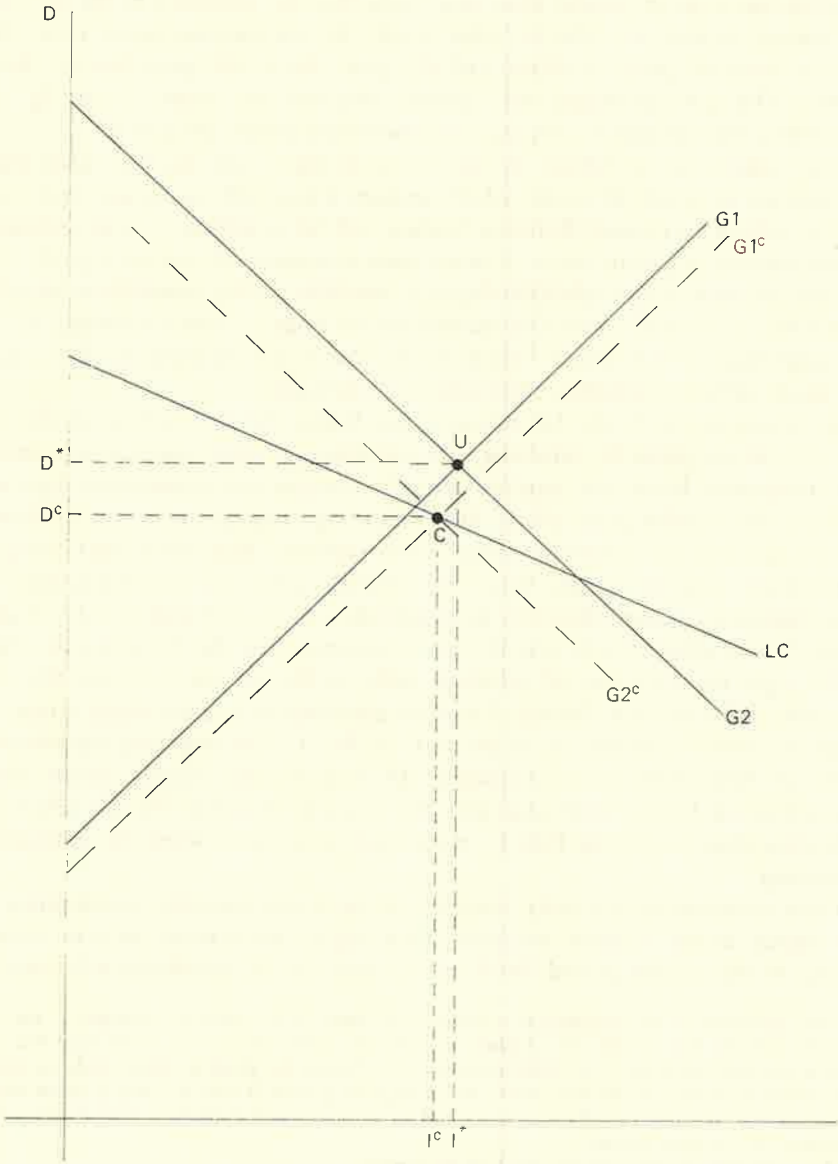

Fig. 2

5. Conclusions

In this paper we have linked two main strands in the literature on small, open economy behavior. The first strand is predominantly concerned with the effects of foreign borrowing on relative prices and intersectoral resource shifts over time, given perfect world capital markets. The second focuses on the effect of credit rationing on intertemporal decisions, but generally neglects intersectoral issues.

Foreign borrowing acts directly as a means of smoothing consumption intertemporally. When this borrowing takes the form of traded goods alone, changes in the relative price of non-tradeables are necessary to generate the resource shifts that permit intrasectoral as well as intertemporal consumption smoothing. In particular, the relative price must decline over time to induce the movement of factors out of the traded goods sector into non-traded production when foreign borrowing is adding traded goods resources, and to induce the reallocation of factors back into tradeables production when repayment obligations are due in the future. When the tradeable goods market affects the future supply of non-tradeables via investment of the tradeable good, then additional channels for consumption smoothing exist. We have described circumstances under which investment dampens relative price movements and have shown that the intertemporal elasticity of consumption substitution and factor intensity assumption play key roles in this process.

A binding lending constraint acts to drive a wedge between the marginal return to capital and the world interest rate. Under the assumption that non-tradeables production is labor-intensive and consumption is highly substitutable across periods, the constraint unambiguously causes the postponement of relatively more consumption of both goods into the future. The relative price of non-tradeables is likely to fall in both periods, though by proportionately more in the first, in order to shift resources into the production of traded goods at the time when borrowing is rationed. Under different parameter assumptions, opposing results cannot be ruled out.

Appendix: Supply functions and factor price expressions

Assuming Hicks-neutral technological progress, the equations relating changes in output (Q_N, Q_T), output prices (p_N, p_T), factor returns (w, ρ), endowments (K, L) and technology (q_N, q_T), can be written in differential form (with time subscripts omitted) as

$$\lambda_{LN}\hat{Q}_N + \lambda_{LT}\hat{Q}_T = \hat{L} + \lambda_{LN}\hat{q}_N + \lambda_{LT}\hat{q}_T + \delta_L(\hat{w} - \hat{\rho}), \tag{A.1}$$

$$\lambda_{KN}\hat{Q}_N + \lambda_{KT}\hat{Q}_T = \hat{K} + \lambda_{KN}\hat{q}_N + \lambda_{KT}\hat{q}_T - \delta_K(\hat{w} - \hat{\rho}), \tag{A.2}$$

$$\theta_{LN}\hat{w}+\theta_{KN}\hat{\rho}=\hat{p}_N+\hat{q}_N, \tag{A.3}$$

$$\theta_{LT}\hat{w}+\theta_{KT}\hat{\rho}=\hat{p}_T+\hat{q}_T, \tag{A.4}$$

where

$$\delta_L=\lambda_{LN}\theta_{KN}\theta_N+\lambda_{LT}\theta_{KT}\theta_T, \qquad \delta_K=\lambda_{KN}\theta_{LN}\theta_N+\lambda_{KT}\theta_{LT}\theta_T.$$

The notation used is that of Jones (1985): a circumflex ($\hat{}$) over a variable indicates its proportional rate of change (e.g., $\hat{Q}_N=dQ_N/Q_N$); λ_{ij} is the proportion of factor i employed in sector j ($i=K, L$; $j=N, T$); θ_{ij} is the share of factor i in the output value of sector j; θ_j is the elasticity of factor substitution in section j; and $\hat{q}_j$ measures the extent of Hicks-neutral technological progress in sector j.

Solving (A.1)–(A.4) for $\hat{Q}_N, \hat{Q}_T, \hat{\rho}$ and $\hat{w}$, while assuming no labor force growth ($\hat{L}=0$), implies

$$\hat{Q}_N=(1+e)\hat{q}_N-e\hat{q}_T+e(\hat{p}_N-\hat{p}_T)-k\hat{K}, \tag{A.5}$$

$$\hat{Q}_T=(1+e_T)\hat{q}_T-e_T\hat{q}_N-e_T(\hat{p}_N-\hat{p}_T)+k_T\hat{K}, \tag{A.6}$$

$$\hat{\rho}=(1+\alpha)(\hat{p}_T+\hat{q}_T)-\alpha(\hat{p}_N+\hat{q}_N), \tag{A.7}$$

$$\hat{w}=-\beta(\hat{p}_T+\hat{q}_T)+(1+\beta)(\hat{p}_N+\hat{q}_N), \tag{A.8}$$

where

$$e=(\lambda_{KT}\,\delta_L+\lambda_{LT}\,\delta_K)/(|\lambda|\,|\theta|),$$

$$e_T=(\lambda_{KN}\,\delta_L+\lambda_{LN}\,\delta_K)/(|\lambda|\,|\theta|),$$

$$k_N=\lambda_{LT}/|\lambda|, \qquad k_T=\lambda_{LN}/|\lambda|,$$

$$|\lambda|=\lambda_{LN}-\lambda_{KN}, \qquad |\theta|=\theta_{LN}-\theta_{LT},$$

$$\alpha=\theta_{LT}/|\theta|, \qquad \beta=\theta_{KN}/|\theta|.$$

Note that $|\lambda|$ and $|\theta|$ are both positive by the assumption that the non-traded sector is relatively labor intensive.

Assuming that the price of traded goods is fixed by world markets ($\hat{p}_T=0$) implies that the relative price of non-traded goods, p, is given by p_N. Expressions (A.5) and (A.6) then imply that the supply functions for non-traded and traded goods may be written in constant elasticity form as

$$Q_N=q_N^{1+e}q_T^{-e}p^eK^{-k}, \qquad Q_T=q_N^{-e_T}q_T^{1+e_T}p^{-e_T}K^{k_T}. \tag{A.9, A.10}$$

Similarly, factor returns may be expressed as

$$\rho = q_{\mathrm{N}}^{-\alpha} q_{\mathrm{T}}^{1+\alpha} p^{-\alpha}, \qquad w = q_{\mathrm{N}}^{1+\beta} q_{\mathrm{T}}^{-\beta} p^{1+\beta}. \qquad \text{(A.11), (A.12)}$$

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