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THE GEOMETRY OF ASSET ADJUSTMENT WITH ADJUSTMENT COSTS

*Reuven Glick**

Abstract

This paper provides a geometric analysis of asset adjustment behavior in response to changes in expected return and wealth. The analysis permits discussion of the role of different adjustment cost and risk attitude assumptions within a unified framework. It is shown that changes in expected return generally induce portfolio revision only if the marginal rate of substitution between assets is less than their relative marginal costs of adjustment. Wealth disturbances may induce sequential, rather than simultaneous, asset adjustment if marginal adjustment costs are constant or decreasing and the investor is risk neutral.

I. Introduction

Adjustment costs and risk-return preferences play important roles in decisions to accumulate or liquidate assets. This paper presents a geometric analysis of one-period asset adjustment behavior in response to changes in expected return and wealth under different adjustment cost and risk attitude assumptions. The analysis permits a synthesis of existing results and provides further insight into the roles of adjustment costs and risk-return preferences in asset adjustment decisions.

A wide variety of financial and economic decision models analyze asset adjustment strategies. Portfolio revision models [e.g., Smith (1967), Chen, Jen, and Zionts (1971), Kamin (1975), Magill and Constantinides (1976), Abrams and Karmarkar (1980)] address the reallocation of financial asset holdings due to expected return and risk changes. The portfolio revision model of Chen, Jen, and Zionts (1974) and inventory models of money demand of Baumol (1952) and Miller and Orr (1966) discuss asset adjustment in response to fluctuating cash needs. Parallel analyses of asset adjustment occur in models of the effect of fluctuating deposit outflows on banks [e.g., Orr and Mellon (1961), Poole (1968), Frost (1971), Baltensperger (1972)] and of demand fluctuations on commodity inventory holdings [Peterson and Silver (1978)].

Portfolio models usually assume risk-averse behavior, while inventory and banking models presume risk neutrality. Most asset adjustment models assume constant marginal adjustment costs. Whereas the latter assumption may be appropriate for analyzing the management of financial assets, real assets such as commodity inventories and physical capital investments are better characterized by increasing marginal adjustment costs. The geometric framework developed here allows for a general analysis of asset adjustment under different adjustment cost and risk attitude assumptions. Within this framework the circumstances under which changes in return or wealth induce either no adjustment, simultaneous adjustment, or sequential adjustment of assets are examined.

*New York University.

The model is formulated in section II. The equilibrium adjustment of asset holdings in response to return and wealth disturbances is described in section III. Section IV contains concluding comments.

II. The Model

Asset management behavior is analyzed for a stylized asset-holding unit over a single period. This asset-holding unit may be a financial institution, such as a bank, mutual fund, or insurance company, a non-financial institution, such as a manufacturing firm, or an individual. For convenience, the asset-holding unit will be referred to as an "investor." Depending on the nature of the investor's activity, the assets held may be financial, such as securities, or real, such as physical capital or commodity inventories. The investor is assumed to possess an initial portfolio containing given amounts of two assets and to experience either a change in expected return or a wealth disturbance of known magnitude at the beginning of the period. The model can be generalized easily to include more assets. Given that asset returns are stochastic, the asset management problem is to choose a new portfolio that maximizes the expected utility of end-of-period wealth.

Let A_i° and A_i ($i = 1,2$) be the amounts of asset i held in the initial and new portfolios, respectively. Short sales of assets are precluded, i.e., $A_i^\circ \geq 0$, $A_i \geq 0$. Let Z represent the *decrease* in wealth experienced at the beginning of the period. This wealth change may arise from a sudden cash need in the case of an individual or firm, a deposit outflow in the case of a financial institution, or an unanticipated depletion of inventories in the case of a manufacturing firm.

Adjustment Costs

The adjustment costs for a change in holdings of asset i between the initial and new portfolios are:

$$C_i = C_i\left[\; | A_i - A_i^\circ | \;\right], C_i' \geq 0, C_i \geq 0, C_i[0] = 0, \text{i} = 1,2. \tag{1}$$

These costs may be transaction fees, real resource costs of purchasing and selling, and/or costs of subjectively valued time and effort. The above specification assumes symmetric costs that are positively related to the *absolute* value of the change in asset holdings, $| A_i - A_i^\circ |$; therefore, the effect on cost of liquidating or accumulating a given amount of a particular asset is equal. Considering asymmetric costs would not affect results.[1]

The marginal effect on adjustment costs of a change in $A_i - A_i^\circ$ is positive ($C_i' > 0$). Financial models incorporating asset adjustment behavior typically assume adjustment costs are proportional ($C_i' = c_i$, $C_i'' = 0$). This is a reasonable assumption if

[1] An alternative approach is to define adjustment costs in terms of the correlation between the disturbances necessitating adjustment and asset returns. See Chen, Jen, and Zionts (1974) and Chen, Kim, and Kon (1975). With this specification, an asset is considered to be more liquid the greater its return as adjustment needs increase. This approach can easily be incorporated into the present model as well. See footnote 2.

brokerage fees, commissions, and other transaction costs are proportional to the amount of an asset bought or sold. If there are transaction economies, however, marginal adjustment costs may decrease with the amount of adjustment ($C_i'' < 0$). Furthermore, in some situations, particularly with real assets, marginal adjustment costs may be increasing ($C_i'' > 0$). For example, the marginal labor and resource costs of increasing or decreasing commodity inventories of different products may rise with the physical amount that is shipped. Similarly, the marginal costs of acquiring or scrapping physical capital may rise with the amount involved. All three assumptions concerning the nature of marginal adjustment costs will be considered below.

Adjustment Constraint

Asset holdings in the new portfolio equal initial holdings *less* the wealth decrease and the adjustment costs incurred in revising the initial portfolio. Therefore, the adjustment constraint relation is:

$$(A_1 - A_1^\circ) + (A_2 - A_2^\circ) = -Z - C_1\Big[\,|A_1 - A_1^\circ|\,\Big] - C_2\Big[\,|A_2 - A_2^\circ|\,\Big]. \qquad (2)$$

Since $C_i > 0$, equation (2) implies that a given initial wealth *decrease* ($Z > 0$) necessitates liquidation of one or both assets ($A_i - A_i^\circ < 0$ for $i = 1$ and/or $i = 2$), while a given initial wealth *increase* ($Z < 0$) enables accumulation of one or both assets ($A_i - A^\circ > 0$ for $i = 1$ and/or $i = 2$). With no wealth change ($Z = 0$), accumulation of one asset requires liquidation of the other ($A_i - A_i^\circ > 0$ implies $A_j - A_j^\circ < 0$ for $i \neq j$).

Objective Function

Assets held in the new portfolio are assumed to yield an end-of-period stochastic rate of return, R_i, per unit of holding of asset i. These returns are assumed to be normally distributed with mean $\bar{R}_i$, variance σ_i^2, and covariance σ_{12}. End-of-period wealth is $W = A_1 (1 + \bar{R}_1) + A_2 (1 + \bar{R}_2)$. The probability distribution of W is thus determined by the distribution of returns and the amounts of the individual assets held in the new portfolio. Assuming a mean-variance utility function, $U[W]$, the investor's decision problem is, given his initial portfolio and knowledge of return distributions and the magnitude of any wealth disturbance, to choose levels of A_1 and A_2 in the new portfolio that maximize

$$E\Big[\,U[W]\,\Big] = E[W] - \frac{b}{2}V[W] = A_1(1 + \bar{R}_1) + A_2(1 + \bar{R}_2)$$
$$- \frac{b}{2}(A_1^2\sigma_1^2 + A_2^2\sigma_2^2 + 2A_1A_2\sigma_{12}) \qquad (3)$$

subject to equation (2),

where

E = the expectation operator,
V = the variance operator, and
b = the investor's (constant) marginal tradeoff between expected return and risk.

Graphical Depiction

Implicit differentiation of expected utility with respect to A_1 and A_2 gives the (absolute-valued) slope of the iso-utility contours in (A_1, A_2) space

$$\frac{dA_2}{dA_1} = \frac{1 + \bar{R}_1 - b(A_1\sigma_1^2 + A_2\sigma_{12})}{1 + \bar{R}_2 - b(A_1\sigma_2^2 + A_1\sigma_{12})}$$

and, hence, the marginal rate of substitution (MRS) of asset 2 for asset 1 in the investor's portfolio.[2] These contours are analogous to the indifference curves of standard portfolio analysis [e.g., Tobin (1958), Sharpe (1970)], which are usually depicted in risk-return space. Assuming positive marginal utility (i.e., $1 + \bar{R}_i - b(A_i\sigma_i^2 + A_j\sigma_{ij}) > 0, i \neq j$), these contours are downward sloping in (A_1, A_2) space. Curves further to the northeast correspond to higher utility. For a risk-averse investor (b > 0), the curves are bowed to the origin as depicted in the three panels of Figure I. For a risk-neutral investor (b = 0), the slope is constant and depends only on relative expected returns $(1 + \bar{R}_1)/(1 + \bar{R}_2)$. The curves become steeper if the relative expected return to asset 1 increases or, for a risk-averse investor, if the relative risk of asset 1 declines.

If there is no wealth disturbance ($Z = 0$) and marginal adjustment costs (MAC) are zero ($C_i' = 0$), then the adjustment constraint [equation (1)] is the straight line ABC in Figure I with slope -1. Any accumulation of one asset requires liquidation of the other by an equivalent amount $(A_1 - A_1^\circ) = -(A_2 - A_2^\circ)$. When MAC are positive, however, accumulation of one asset requires liquidation of the other by a greater amount. Consequently, the adjustment relation is a kinked opportunity locus—DBF—which (except for point B) lies inside the region defined by the zero adjustment cost line ABC.

Implicit differentiation of total costs $C = C_1 + C_2$ with respect to A_1 and A_2 gives the (absolute-valued) slope of this locus:

$$\frac{dA_2}{dA_1} = \begin{cases} \dfrac{1 - C_1'}{1 + C_2'} \text{ for } A_1 - A_1^\circ < 0, A_2 - A_2^\circ > 0 \text{ (segment DB)} \\ \\ \dfrac{1 + C_1'}{1 - C_2'} \text{ for } A_1 - A_1^\circ > 0, A_2 - A_2^\circ < 0 \text{ (segment BF).} \end{cases}$$

As shown in the three panels of Figure I, the opportunity locus is bowed-out, linear, or bowed-in as MAC are either increasing ($C_i'' > 0$), constant ($C_i'' = 0$), or decreasing (C_i''

[2] The framework can be extended easily to incorporate end-of-period stochastic wealth disturbances by defining $W' = W - L$, where L represents (random) end-of-period cash demand. The investor then maximizes

$$E[U[W - L]] = E[U[W]] - \bar{L} - \frac{b}{2}(\sigma_L^2 - 2A_1\sigma_{1L} - 2A_2\sigma_{2L}),$$

where $L \sim N(\bar{L}, \sigma_L^2)$ with covariance σ_{1L} and σ_{2L} with A_1 and A_2, respectively. The iso-utility curves have slope

$$\frac{dA_2}{dA_1} = \frac{\bar{R}_1 - b(A_1\sigma_1^2 - \sigma_{1L} + A_2\sigma_{12})}{\bar{R}_2 - b(A_2\sigma_2^2 - \sigma_{2L} + A_1\sigma_{12})}.$$

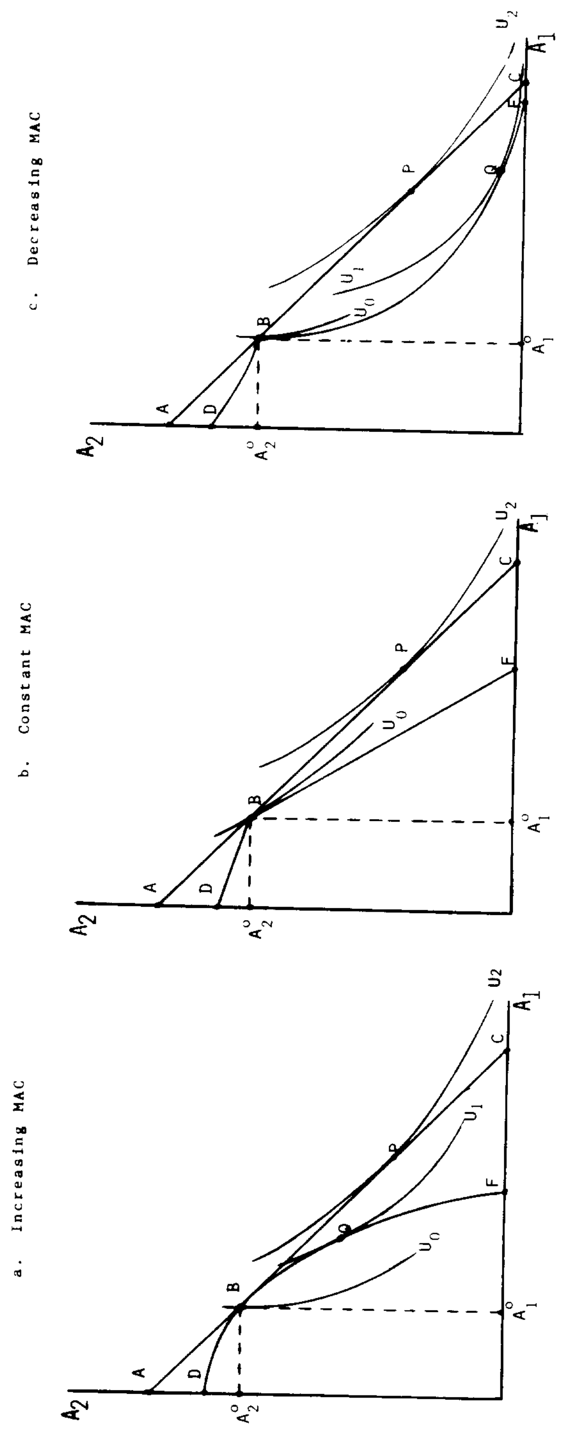

Figure I. Portfolio Revision in Response to an Expected Return Change.

< 0). When holdings of each asset in the new portfolio equal that in the initial portfolio, as at point B, neither asset is adjusted and, hence, total adjustment costs are zero. Movements to the northwest (southeast) of B along the segment DB (BF) result in adjustment costs and imply liquidation (accumulation) of asset 1 and accumulation (liquidation) of asset 2.

As shown in Figure II, a wealth decrease ($Z > 0$) results in shifting the opportunity locus DBF inward to the position GHIJ. A decrease in wealth requires that for any level of adjustment of one asset, more (less) of the other must be liquidated (accumulated). Now the locus has two kinks, at points H and I. The segment GH describes points where asset 1 is liquidated and asset 2 accumulated, while the segment IJ describes points where asset 1 is accumulated and asset 2 is liquidated. The slopes of these segments are the same as that for DB and BF, respectively. The segment HI describes points where *both* assets are liquidated in response to the wealth decrease. The slope of HI is

$$\frac{dA_2}{dA_1} = \frac{1 - C_1'}{1 - C_2'} \text{ for } A_1 - A_1^\circ < 0,\ A_2 - A_2^\circ < 0 \text{ (segment HI).}$$

Analogously, in the case of a wealth increase ($Z < 0$), the opportunity locus shifts outward from DBF and there exists a segment with slope

$$\frac{dA_2}{dA_1} = \frac{1 + C_1'}{1 + C_2'} \text{ for } A_1 - A_1^\circ > 0,\ A_2 - A_2^\circ > 0$$

along which both assets are accumulated.

III. Determination of Optimal Asset Management Strategy

The optimal asset management strategy is composed of the asset levels in the new portfolio (A_1^*, A_2^*) that maximize expected utility, equation (3), while satisfying the adjustment constraint, equation (2). Graphically, the optimal solution is given by the point on the opportunity locus that touches the iso-utility curve representing the highest possible level of expected utility. The location of this point and, thus, the nature of the solution will depend on the initial asset holdings (A_1°, A_2°), the magnitude of decrease in wealth (Z), the nature of adjustment costs, and the investor's attitude toward risk. The circumstances under which portfolio revision occurs due to a change in expected returns is addressed first. Subsequently, the effect of a wealth disturbance is analyzed.

Portfolio Revision with a Change in Expected Return

In the absence of a wealth disturbance ($Z = 0$) and of any adjustment costs ($C_i = 0$), the desired portfolio of a risk-averse investor occurs at the tangency between an iso-utility curve and the line ABC. This tangency corresponds to the equilibrium result of the standard portfolio models of investment [e.g., Markowitz (1959), Tobin (1958), and Sharpe (1970)] and bank asset management [e.g., Parkin (1970)].

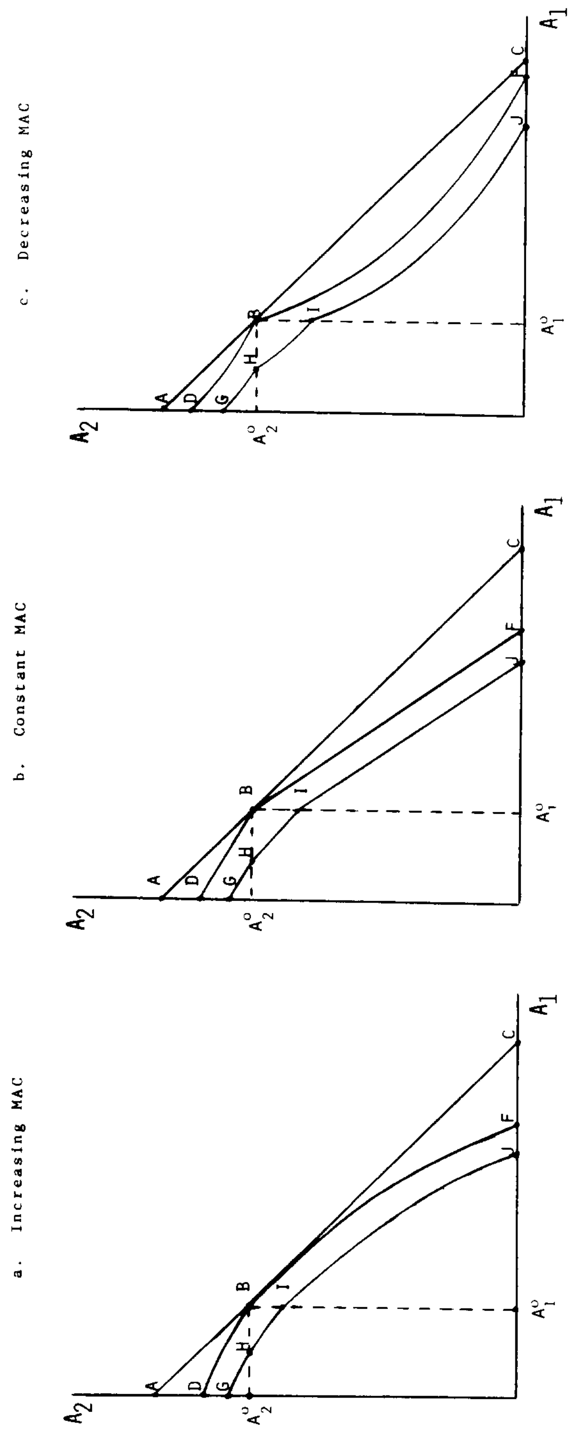

Figure II. Portfolio Revision in Response to a Wealth Decrease.

Assume that the beginning portfolio, given by point B, initially corresponds to such a point of tangency. Consider an increase in $\bar{R}_1$, the expected return to asset 1. The iso-utility curves become steeper. In the absence of adjustment costs, the investor will move to the new point of tangency (point P in Figure I) and revise his portfolio to include more of asset 1. With adjustment costs, the portfolio given by point P is infeasible since it lies beyond the opportunity locus DBF. Whether the investor can increase his utility by revising his portfolio now depends on the marginal costs of adjustment as well as the marginal rate of substitution between assets.

In the cases of constant or increasing MAC, the investor will prefer to hold the initial portfolio at B if, in the "neighborhood" of point B, the slope of the iso-utility curve through B is steeper than the locus DB and flatter than the locus BF, or equivalently if[3]

$$\frac{1 - C_1'[0]}{1 + C_2'[0]} < \frac{1 + \bar{R}_1 - b(A_1^\circ \sigma_1^2 - A_2^\circ \sigma_{12})}{1 + \bar{R}_2 - b(A_2^\circ \sigma_2^2 - A_1^\circ \sigma_{12})} < \frac{1 + C_1'[0]}{1 - C_2'[0]} . \qquad (5)$$

Thus, the investor will not revise his portfolio if the marginal rate of substitution (MRS) of asset 2 for 1 is less than the relative MAC of accumulating asset 1 and liquidating asset 2 and greater than the relative MAC of liquidating asset 1 and accumulating asset 2. This case is illustrated in panel b of Figure I. Changes in expected return or risk will only induce a change in portfolio holdings if they cause the MRS to lie outside the bounds defined by condition (5). In other words, portfolio revision occurs only if the relative marginal benefits exceed the associated relative marginal adjustment costs. This case is illustrated in panel a of Figure I: the investor chooses the new portfolio given by point Q, where he attains a level of utility higher than with his initial portfolio at point B, but necessarily less than at P. Attainment of this new portfolio involves simultaneous liquidation of asset 2 and accumulation of asset 1.

In the case of decreasing MAC, portfolio revision may take place even if condition (5) is satisfied. In that case, while small asset adjustments from their initial levels may be undesirable, large adjustments may sufficiently lower total adjustment costs to justify portfolio revision. This is illustrated in panel c of Figure I where the revised portfolio Q is preferred to the initial one, portfolio B, even though the iso-utility curve through B is flatter than BF at that point.

The above condition for portfolio revision has been derived in the existing literature in various forms. For example, Chen, Jen, and Zionts (1971) and Abrams and Karmarkar (1980) arrive at a similar condition for portfolio revision for a risk-averse investor with constant MAC. Frost (1971) and Baltensperger (1972) find an analogous condition for reserve management in the case of a risk-neutral commercial bank with constant MAC. The analysis here shows that this result holds under more general assumptions concerning the nature of adjustment costs and attitude toward risk.

The analysis presented here also has interesting implications for the circumstances under which an investor maintains a diversified portfolio. It is well known from standard portfolio analysis that a risk-averse investor chooses to diversify while a risk-

[3] This condition is obtained by evalutating the derivatives of the iso-utility curves and MAC line "near" B, i.e., for $A_i - A_i^\circ = 0$.

neutral investor does not. In the former case, decreasing MRS between assets limits the desirability of holding too much of any one asset. In the latter case, the MRS is constant and only the asset with the highest expected return is held. With one important exception, these results hold for the framework presented here. A risk-averse investor who revises his portfolio will remain diversified irrespective of the nature of the adjustment costs incurred. A risk-neutral investor who holds an initially diversified portfolio, but finds it desirable to revise, will prefer to hold a new portfolio that is undiversified when MAC are either constant or increasing. When MAC are increasing, however, a risk-neutral investor will usually prefer a new portfolio that remains diversified in order to minimize adjustment costs. Thus, for example, when the costs of scrapping and replacing less productive real capital equipment are increasing, risk-neutral firms may continue to utilize capital of differential productivity.

Portfolio Revision with a Wealth Disturbance

Now consider the effect of a wealth decrease ($Z > 0$) on the investor's new desired portfolio. It will be shown that under some circumstances it is optimal to liquidate all of one asset before adjusting the other asset, i.e., asset adjustment occurs sequentially. Under other circumstances, it is optimal to liquidate all assets simultaneously. The analysis of a wealth increase is analogous and is not discussed.

Assume that in the absence of a wealth disturbance, condition (5) is satisfied and the investor is content to hold his initial portfolio, given by point B. As discussed in section II and illustrated in Figure II, a wealth decrease shifts the opportunity locus inward. The curvature properties of the iso-utility curves imply that the new equilibrium following a wealth decrease cannot be located on either the segment GH or IJ and the investor will never choose to increase holdings of either asset. The equilibrium, thus, will be somewhere on the segment HI.

Two types of solution are possible. The first is an interior solution somewhere between points H and I, which implies that both assets are liquidated. This solution may arise either for a risk-averse investor irrespective of the nature of MAC or for a risk-neutral investor facing increasing MAC. Under these circumstances, it is optimal for the investor to adjust both assets simultaneously until MRS equal MAC.

The second type of solution is a corner solution that occurs at either endpoint H or I, which implies only one asset is liquidated. A corner solution may arise if MAC are either constant or decreasing. This case may be better understood by referring to Figure III, where risk neutrality and constant MAC are assumed. In the situation depicted there, the constant-sloped iso-utility curves are all flatter than the segment HI, but steeper than the segment GH:

$$\frac{1 - C_1'}{1 + C_2'} < \frac{1 + \bar{R}_1}{1 + \bar{R}_2} < \frac{1 - C_1'}{1 - C_2'}$$

The relative expected return to holding asset 1 is less than the relative cost of liquidating it. The new desired portfolio is given by point H, implying that the investor should accommodate the wealth decrease by liquidating only asset 1 and leaving holdings of asset

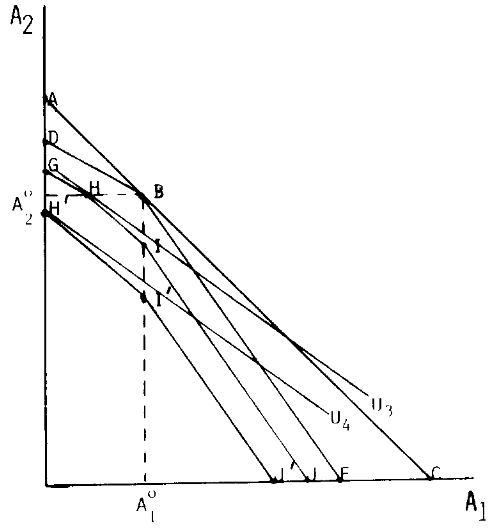

Figure III. Portfolio Revision to a Wealth Decrease for a Risk-Neutral Investor with Constant MAC.

2 unchanged. The asset chosen for liquidation depends on both its relative expected return and its relative MAC. For example, even if the relative marginal costs of selling asset 1 rather than asset 2 are high, it is still optimal to liquidate asset 1 if the relative expected returns to holding it are sufficiently low.

The analysis above implicitly assumes that the magnitude of the wealth decrease does not exceed the initial holdings of either asset ($A_1^o > Z$). If this were not so, it would be necessary to liquidate holdings of both assets even when the investor is risk-neutral and MAC are constant (or decreasing). This may be better understood by again referring to Figure III. The segment $H'I'J'$ lying to the southwest of $GHIJ$ describes the opportunity locus for a wealth decrease that exceeds A_1^o. The optimal solution occurs at point H'. Liquidating asset 2 is now also required since, given the inability to sell assets short, A_1^o is the maximum amount of asset 1 that can be liquidated. All of asset 1 is liquidated first and the residual wealth decrease is accompanied by sequentially liquidating asset 2.

The property of sequential asset adjustment is characteristic of many banking models [Orr and Mellon (1961), Poole (1968), Frost (1971), Baltensperger (1972)]. In these models wealth disturbances take the form of fluctuating deposit holdings. Assuming risk neutrality and constant MAC, they usually presume that one asset, such as excess reserves, is used first to accommodate deposit outflows, followed, if necessary, by the sale of securities or recall of loans.

IV. Summary and Conclusions

This paper presents a geometric analysis of the roles that adjustment costs and risk attitudes play in asset adjustment strategy. It demonstrates that changes in expected return (or risk) generally do not induce portfolio revision if the marginal rate of substitution between an individual asset and any others in the portfolio is less than its relative marginal costs of adjustment. An exception to this occurs if marginal adjustment costs are sufficiently decreasing. The analysis also shows that, if the investor is risk-averse or if the investor is risk-neutral and marginal adjustment costs are increasing, then a wealth disturbance generally prompts simultaneous adjustment of all assets until their marginal rates of substitution and relative adjustment costs are equalized. If, however, the investor is risk neutral and marginal adjustment costs are constant or decreasing, then assets will be sequentially adjusted.

Adjustment costs in this analysis are assumed to vary with the change in asset holdings. One possible extension is to incorporate costs of adjustment that are independent of the magnitude or direction of change in asset holdings. In the case of a wealth disturbance, the existence of such fixed costs necessitates a comparison of the costs of simultaneously adjusting and incurring fixed costs on all assets with the variable and fixed costs of adjusting only a single asset. This suggests the possibility of sequential asset adjustment even when investors are risk averse and/or marginal adjustment costs are increasing.

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